Unlisted Infrastructure Debt Performance Measurement

Frédéric Blanc-Brude[a,b,1,*], Majid Hasan[a,b], Omneia R.H. Ismail[a,b]

[a] *EDHEC Business School, 393 Promenade des Anglais, F-06202 Nice Cedex 3, France*

[b] *#07-02, 01 George Street, EDHEC Risk Institute-Asia, 049145, Singapore*

Abstract

In this paper, we develop the first structural valuation framework for unlisted infrastructure project finance (PF) debt. PF is a unique form of corporate governance, which creates extensive control rights for lenders through covenants and embedded options and existing valuation methodologies cannot be directly applied to such loans. Our methodology is designed to require a parsimonious dataset of observable inputs, and provides a clear link between an infrastructure project's fundamentals and the risk profile of its senior debt. For reasonable parameter estimates the model reproduces the known observed probabilities of default and recovery rates of project finance loans, but also provides investors and regulators with performance metrics unavailable to them until now, including a measure of the impact of the frequent restructurings of infrastructure project debt, of their dynamic risk profile, and of the trade-off between duration and credit risk.

Keywords: Infrastructure, Project finance, Credit risk, debt renegotiation, Incomplete markets, Risk-neutral

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1. Introduction

Both long-term investors and prudential regulators have become increasingly aware of growing investment opportunities in illiquid infrastructure debt in recent years. However, a valuation

*Corresponding author

Email addresses: frederic.blanc-brude@edhec.edu (Frédéric Blanc-Brude), majid.hasan@edhec.edu (Majid Hasan), omneia.ismail@edhec.edu (Omneia R.H. Ismail)

[1] Telephone No: +65 6438 0030; Fax No: +65 6438 9891; Postal address: #07-02, 01 George Street, EDHEC Risk Institute-Asia, 049145, Singapore

framework allowing the derivation of adequate return and risk measures for this type of instruments has remained elusive. A specific valuation framework is needed, first because infrastructure project debt has unique characteristics not found in standard corporate credit instruments, and second because illiquidity implies a high degree of data paucity, which needs to be explicitly addressed in the valuation approach.

Infrastructure projects are typically carried out through non-recourse Project Finance (PF), which entails establishing a Special Purpose Entity (SPE) that is financed in large part by private bank loans. Therefore, when discussing PF debt, we focus exclusively on PF loans since it is the most widely used and representative type of credit instrument used to finance infrastructure projects.

Most existing research on project loans is of empirical nature. Kleimeier and Megginson (2000) and Sorge and Gadanecz (2008) study the term structure of PF loan spreads. They find a humped shaped relation between loan maturities and credit spreads, and observe lower credit spreads for PF loans as compared to corporate loans. Blanc-Brude and Strange (2007) and Blanc-Brude and Ismail (2013) study the determinants of credit spreads in project finance loans. Such empirical work has so far remained limited to the attributes of project finance debt at the time of financial close, because such loans are essentially not traded and a panel database of cash flows spanning the entire life of a representative sample of infrastructure projects does not exist today. We return to this point below.

Existing studies do not document the evolution of credit risk over the life of PF loans. Credit risk dynamics are partially addressed in studies conducted by rating agencies, which aim to observe events of defaults and recovery rates within a population of loans at a given point in time in a the lifecycle of infrastructure projects — from the construction stage, to 20 or 30 years of operation and maintenance. However, these studies rely on samples that can be large in the cross section but still do not span the entire life of infrastructure projects. Reported metrics typically cover the first ten years of the project lifecycle. (see for example Moody's, 2012, 2013, 2014; Standard and Poor's, 2013)

Several stylised facts are frequently abstracted from Rating Agency reports:

- On average, the available sample of project finance loans exhibits decreasing marginal default rates in time;
- As a consequence, these loans exhibits a continuous credit risk transition over a period of

approximately ten years, from approximately a triple-B equivalent to a single-A or better equivalent. For example, the observed probability of default in Moody's sample trends form around 2% around the time of financial close, to near zero after ten years;

- We also note here that the same studies also attempt to isolate so-called public-private partnership (PPP) projects that mostly receive a contracted income stream from the public sector and finds that their probability of default is lower and averages about 0.5% across their entire lifecycle.

These results may be affected by sampling biases (see Blanc-Brude and Ismail, 2013) and lack any clear identification of the relationship between credit risk and loan or project characteristics. In addition, they do not take into account the time evolution of some important risk measures such as expected loss, value-at-risk (VaR) or expected shortfall (conditional VaR). Hence, they fail to provide investors with a view of the full distribution of losses over time, despite reporting aggregate recovery rates. Still, they are informative and provide us with an empirical point of comparison to the output of our model, which we discuss in section 3.

In terms of valuation framework, models commonly used in the financial industry to value PF and corporate loans include capital budgeting models that determine a project's feasibility under a base case scenario by calculating its NPV, IRR, payback period etc. These models are, however, static in nature, ignore the effects of debt covenants and embedded options, and fail to shed any light on the evolution of the credit risk profile in time. Such models often assume a constant loss given default (LGD, and a constant credit spread. Both these variables, however, should depend on the underlying risk profile of the project, and should change with time as its risk profile changes.

A more sophisticated approach used by banks to measure the risk-based performance of loans is the Risk Adjusted Return on Capital or RAROC: the ratio of the adjusted income from the loan to a risk-based capital requirement, that is, the amount of capital needed to limit total default probability to a certain level weighted by the marginal contribution of the loan to total loss for the bank. The decision to lend is made if the RAROC exceeds a required hurdle rate (see Shearer and Forest Jr, 1997; Froot and Stein, 1998; Aguais et al., 2000).

This approach has several limitations, chief amongst which is that it is concerned only with the losses that may lead to default and ignores smaller but more probable losses. The RAROC measure is also insensitive to the structure of the security (type of loan, amortisation of principal, covenants, collateral requirements, repayment rights, etc) and embedded options, and often uses the bank's internal cost of capital as a hurdle rate, which is unlikely to be the same than other investors.

The academic literature on PF loan valuation is very thin. Multinomial tree-based option pricing models have also been applied to PF debt (see for example Ho and Liu, 2002; Wibowo, 2009) and while these methods can take into account some debt covenants, they fail to incorporate the endogenous nature of credit risk, and cannot be used when cash flows are path dependant. Other loan models include Dunsky and Ho (2007) or Aguais and Santomero (1998), which construct empirical regression models to value mortgages and corporate loans. However, as argued before, due to the lack of market prices or full panel series of project finance loan cash flows, such approaches cannot be applied to PF loans.

In this paper, we remedy these shortcomings by developing the first theoretical valuation and risk measurement framework for unlisted infrastructure project debt instruments that derives PF loans' credit risk profile from the project company's fundamentals and its loan covenants using a minimum amount of input data. Moreover, our model incorporates the heterogeneity of investors' risk preferences due to the illiquid nature of PF loans. Thus, our proposed valuation framework is relevant not only to form risk-return expectations, but also for designing loan structures to achieve a required risk profile, and setting regulatory requirements for investors in PF loans.

1.1. Objectives of this paper

The objectives of this paper are:

1. To determine the most appropriate pricing model for infrastructure project finance loans;
2. To design a methodology that can be readily applied given the current state of empirical knowledge and at a minimum cost in terms of data collection;
3. To derive the most relevant return and risk measures for long-term debt investors and regulators: *per period loss, value-at-risk (VaR), expected shortfall or conditional VaR (CVaR), expected recovery rates, duration, yield, and z-spread*;
4. To define the minimum data collection requirements for infrastructure project loan valuation that can nevertheless inform a robust pricing model.

1.2. Structure of this paper

The rest of this paper is structured as follows: chapter 2 discusses the definition and characteristics of infrastructure project finance debt. Chapter 3 provides an overview of our valuation approach. Chapter 4 details the implementation of the model for two generic types of infrastructure projects. In chapter 5, we present the resulting risk and return measures of illiquid infrastructure project debt obtained for the two generic types of infrastructure projects. Chapter 6 discusses our findings.

2. Definition and Characteristics of Infrastructure Debt

In this section, we provide a justification of our choice of definition of underlying infrastructure debt, and discuss its main characteristics.

2.1. Definition of Inrastructure debt

What constitutes "infrastructure" remains to be universally defined, and any definition of *infrastructure debt* is a matter of trade-off between clarity and comprehensiveness. Our proposed choice is first determined by the requirement to have a clear definition of the underlying in a context where, because of data paucity, we must rely on *ex-ante cash flow models* that can later be calibrated to available empirical observations.

Because infrastructure project finance is well-defined since Basel-II,[2] it provides us with an uncontroversial setting to model expected cash flows, using input parameters for generic project financing structures which are transparent and can be the object of a consensus.

Our focus on project finance as the representative form of infrastructure debt is also warranted because most infrastructure investment and the immense majority of new or 'greenfield' infrastructure projects are delivered via project financing, and private loans constitute the lions' share

[2] "Project finance is a method of funding in which investors look primarily to the revenues generated by a single project, both as the source of repayment and as security for the exposure. In such transactions, investors are usually paid solely or almost exclusively out of the money generated by the contracts for the facility's output, such as the electricity sold by a power plant. The borrower is usually a Special Purpose Entity that is not permitted to perform any function other than developing, owning, and operating the installation. The consequence is that repayment depends primarily on the project's cash flow and on the collateral value of the project's assets." BIS (2005)

of total infrastructure project debt (Yescombe, 2002). Indeed, bond financing has always played a minimal role in project finance globally. In North America, the market in which project bonds are the most used, cumulative issuance between 1994 and 2013 amounts to a mere 5% of the total deal flow.[3]

Hence, by focusing on project finance debt i.e. unlisted senior loans extended to special purpose entities (SPEs) on a limited recourse basis, we capture the bulk of private infrastructure financing and gain a clear definition of infrastructure debt as an underlying instrument. This is instrumental since our purpose is to discuss infrastructure investment on a scale that is congruent with institutional investing i.e. implying substantial asset holdings.

Next, we describe some of the key characteristics of project finance debt.

2.2. Characteristics of Infrastructure Project Finance Loans

2.2.1. Observable Asset Value

In project financing, as opposed to traditional corporate finance, the free cash flow of the firm is the sole determinant of asset value. At any time t during the SPE's finite life, the firm's value is simply the sum of expected net operating cash flow or *cash flow available for debt service* (CFADS), discounted at the appropriate rate. This value is the only quantity against which the SPE may initially borrow (or later re-structure or re-finance) any debt.

In the majority of cases, the project SPE does not own any tangible assets,[4] or owns assets that are so *relationship-specific* that they have little or no value outside of the contractual framework that determines the future CFADS stream, and justifies the investment in the first place.

The only form of collateral available to lenders is known as the loan's "tail" i.e. the SPE's cash flow available for debt service beyond the original maturity of the loan, and over which lenders have control rights in states of the world embodied by certain covenant breaches.

Hence, unlike traditional firms, the value of the total assets of an SPE can be observed. This makes structural credit risk models, which derive a firm's credit risk from its total asset value, a

[3]Source: Dealogic 2014

[4]In the most frequent case of public infrastructure projects financed through a so-called public-private partnership contract, the ownership of the tangible infrastructure assets remains *de facto* and, most often, *de jure* in the public domain.

natural choice.

2.2.2. Covenants, Embedded Options and Restructurings

Because project finance SPEs typically have a high initial leverage (Blanc-Brude and Ismail, 2013; Esty and Megginson, 2003), debt contracts contain numerous covenants to protect debt holders. Common PF debt covenants include "lock up" accounts that block equity dividends if the project's debt service cover ration or DSCR[5] falls below a pre-specified threshold, "cash sweeps" that distribute excess free cash to debt holders, or minimum DSCR requirements that trigger technical defaults if the DSCR falls below a certain level, and allow debt holders to restructure the debt (see Yescombe, 2002).

Thus, PF creditors have significant and extensive control rights through embedded options and debt covenants. For example, debt covenants prohibiting equity holders from raising more cash through new debt or equity issuance to service existing debt can be expected to impact the default mechanism. Likewise, debt holders' right to either restructure infrastructure project debt upon default or liquidate or sell the project company, can have a significant influence on expected recovery rates and the risk/return profile of PF debt. Moreover, PF debt is typically restructured following a breach of covenant, which allows creditors to maximise their *ex post* recovery rate using the loan's tail defined above.

We note that such potential reorganisations of the debt schedule upon default imply that credit risk should be modelled as an endogenous phenomenon.

2.2.3. Knowledge of the Default Point

The default point is more straightforwardly known in PF than in standard corporate finance.

In structural models of standard corporate debt, default is generally modelled as crossing a threshold point below which the total value of the firm's assets is less than its short and medium term liabilities. This is because as long as the total value of the firm is higher than its near term liabilities, equity holders can raise more cash by issuing new equity or debt, and satisfy their current debt obligations. For PF SPEs, this is not the case because equity holders are constrained

[5]The debt service cover ratio at time t is the ratio of the cash flow available for debt service to the debt service at that time.

in their ability to raise more cash by issuing new debt and equity to preserve the value of existing debt holders' security (see Yescombe, 2002, sections 13.7 and 13.10). The non-recourse nature of the equity investment and the inability of the firm to increase its borrowing thus make default easier to predict than in standard corporate finance.

The relationship between the firm's CFADS and the expected senior debt service is captured by a **debt service cover ratio** (DSCR), a quantity that is routinely monitored by project finance lenders. The DSCR at time t is written:

$$DSCR_t = \frac{CFADS_t}{\mathrm{DS}_t^{\mathrm{BC}}} \tag{1}$$

in each period t=1,2,..T for a project financing of maturity T; $\mathrm{DS}^{\mathrm{BC}}$ is the debt service in the "base case", that is, in the original debt contract signed at financial close or $t = 0$.

Thus, a "hard" default of the SPE i.e. an actual default of payment, can be defined in terms of *ex post* $DSCR_t$ as:

$$\mathrm{Default}_t \iff DSCR_t \equiv \frac{CFADS_t}{\mathrm{DS}_t^{\mathrm{BC}}} < 1 \tag{2}$$

However, PF debt covenants also impose other obligations on the borrower in addition to debt repayment and create the possibility of *technical* defaults, which are the prevalent form of credit event in project finance. A "technical" or soft default can be defined as

$$\mathrm{Default}_t \iff DSCR_t \equiv \frac{CFADS_t}{\mathrm{DS}_t^{\mathrm{BC}}} < 1.x. \tag{3}$$

We note that as a function of the CFADS i.e. the underlying process explaining firm value, **the distribution of the DSCR at time t ($DSCR_t$) in project finance can capture both expected asset values and volatility**. Moreover, the DSCR provides an *unambiguous* definition of the default point, in contrast with corporate finance models, in which the actual point of default is harder to pinpoint. In PF loans, if $DSCR_t < 1$, the firm *must* default.

2.2.4. Illiquidity and Lumpiness

Finally, our project finance debt valuation framework must integrate certain contingent features of these instruments, namely illiquidity and size.

A direct consequence of illiquidity is the significant transaction costs associated with buying or selling such instruments, and the absence of time series of market prices for PF debt. The presence of transaction costs makes models built on the assumption of frictionless markets unsuitable for PF debt, while the lack of market prices makes so-called reduced form models of credit risk, which rely on observed market prices, inadequate for pricing PF debt. Such models could be employed if *comparable* traded debt securities existed but because of the many idiosyncratic features of PF loans, this is unlikely to be the case.

Markets for unlisted infrastructure debt thus tend to be both incomplete and not frictionless because of these instruments' illiquidity and lumpiness. This is likely to lead to divergent investor valuations determined in part by risk preferences and by the size of the infrastructure debt allocation in their respective portfolios. Hence, a valuation model of unlisted infrastructure loans must incorporate the existence of upper and lower bounds on value i.e. the absence of a single market price for a given instrument.

Due to these unique characteristics, corporate debt valuation models cannot be directly applied to PF debt. In the next section, we present our valuation approach that incorporates these characteristics.

3. Valuation Approach

In light of the PF loan characteristics discussed in the previous section, we identify four key requirements for our valuation framework

- The model should not rely too heavily on observed market prices of PF loans, and instead should derive loan values from the underlying CFADS process;
- The default boundary should be treated endogenously;
- The model should incorporate path dependant cash flows driven by embedded options;
- The model should incorporate market incompleteness by allowing the valuation to depend on investors' preferences.

These characteristics are best incorporated in structural models of credit risk.

Investor preferences and valuation bounds can be incorporated in structural models using the approximate arbitrage models developed by Cochrane and Saa-Requejo (2000) and Bernardo and Ledoit (2000). The path dependant nature of cash flows can be accounted for by using

Monte Carlo simulations. Examples of structural models that treat the default boundary endogenously include Mella-Barral and Perraudin (1997) and Anderson and Sundaresan (1996). Such endogenous credit risk models, however, assume that renegotiation is costless, that one of the stakeholders have the power to make take-it-or-leave-it offers, and ignore debt covenants. In the case of PF loans, Esty and Megginson (2003) argue that renegotiation costs may play an important role in deterring strategic defaults, that debt covenants play a significant role in determining bargaining power, and that bargaining power is also likely to change over the life of the loan as the project company de-leverages.

Therefore, our valuation framework consists of the following components:

1. We first build a model of $CFADS_t$ using current knowledge of DSCR dynamics and the base case debt schedule.
2. We risk-neutralise the $CFADS_t$ distribution by imposing approximate arbitrage bounds of $[0, 2]$ on investors' required Sharpe ratio.
3. We determine the present value of the debt schedule under the risk neutral measure using the Black Cox decomposition.
4. Finally, to model changes in the debt schedule following any credit event, we take a game theoretic approach to determine the outcome of renegotiation between debt and equity holders with both parties acting in their self-interest.

Next, we outline each step of the valuation framework.

3.1. Cash Flow Dynamics

The first step in our valuation framework is to project the CFADS of the SPE in every state of the world.

3.1.1. The Role of $DSCR_t$

While it is part of *ex ante* cash flow modelling, project CFADS is not necessarily known or monitored *ex post*. However, this measure can be inferred from the debt service cover ratio (DSCR), which is monitored by lenders in project finance, mostly because of its role as a technical default trigger.

Using the definition of $DSCR_t$ given in equation 1, the CFADS for a given period is obtained as:

$$CFADS_t = DSCR_t \times \mathrm{DS}_t^{\mathrm{BC}} \tag{4}$$

with $\mathrm{DS}_t^{\mathrm{BC}}$, the base case debt service defined at financial close. The same relationship holds in expectation.

In other words, as long as the base case debt service is known, we can reduce the question of modelling the free cash flow of the firm in project finance to that of the dynamics of $DSCR_t$ and its determinants.

3.1.2. DSCR Families

With significant data paucity in time series and in the cross-section of infrastructure projects, as discussed in section 1 and in Blanc-Brude (2014), we cannot hope to observe sufficiently large and representative samples of DSCR observations to determine the characteristics of DSCR dynamics empirically. Instead, we must make *a priori* choices about sub-groups of project financial structures, which we expect to correspond to reasonably *homogenous* DSCR dynamics.

In other words, our objective is to partition the infrastructure project finance universe into a parsimonious set of tractable cash flow models, which can be calibrated using available data in due course (e.g. using Bayesian inference techniques). As discussed above, part of the objectives of this paper is to define exactly what data must be collected for this purpose.

In this paper, we focus on two generic groups of infrastructure project finance structures, each of which represents an ideal-type corresponding to numerous existing infrastructure projects. Thus, at financial close, infrastructure projects are typically structured **either with a rising or a flat base case DSCR profile and a more or less long "tail."**

A rising DSCR profile exhibits both a rising mean and implies an increasing volatility of $DSCR_t$. That is, creditors demand a higher DSCR in the future to protect themselves against rising conditional volatility of CFADS. Such projects also have longer "tails" and exhibit between 70% and 80% of initial senior leverage. Projects that are exposed to market risk, such as a power plant that sells electricity at market prices or a toll road, are structured to have a rising DSCR profile. We refer to these projects as *Merchant* infrastructure.

We model the rising DSCR profile using a lognormal distribution with a constant mean and standard deviation, i.e.

$$\frac{d(DSCR_t)}{DSCR_t} = \mu dt + \sigma dW_t, \tag{5}$$

Conversely, a flat DSCR profile exhibits a constant mean and implies constant conditional cash flow volatility. Projects with little to no market risk are structured with a flat DSCR. They also have shorter tails and a higher level of senior leverage, usually around 90%. Moreover, contrary to projects with a rising DSCR, which effectively de-leverage as their lifecycle unfolds, projects with a constant DSCR stay highly leveraged until the end of the debt's life (otherwise their DSCR would rise). Examples of these projects include social infrastructure projects, such as schools or hospitals that receive a fixed payment from the public sector. We refer to these projects as *Contracted* infrastructure.

We model the flat DSCR profile as a normal distribution with a constant mean and standard deviation, i.e.

$$DSCR_t = E[DSCR] + \sigma(DSCR)dW_t. \tag{6}$$

We note that other generic models of project finance structures can be described, not least a hybrid version of the two cases discussed above. However, the Merchant and Contracted cases provide a sufficiently rich set to illustrate our methodology.

3.2. Risk neutralisation

In this section, we show that knowledge of the dynamics of $DSCR_t$ in project finance is sufficient to derive the necessary input of a Merton model of credit risk.

3.2.1. Distance to Default

In the Merton model (Merton, 1974), the firm's assets are assumed to follow a log-normal process with a constant mean and volatility, and the physical probability of default is given by

$$p(t,T) = N\left(\frac{\ln(\frac{A_t}{D}) + (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right), \tag{7}$$

where $p(t,T)$ is the cumulative probability of default between time t to T, A_t is the value of the firm's assets at time t, D is the default threshold, and μ and σ are the mean return and volatility of firm's assets.

Drawing from the Merton model, the KMV model (Crosbie and Bohn, 2003) defines the negative of the quantity inside the brackets as the Distance to Default (DD)

$$DD_T = -\frac{\ln(\frac{A_t}{D}) + (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}. \tag{8}$$

The default probability is the area under the distribution above the DD point

$$p(t,T) = N(-DD_T). \tag{9}$$

The distance to default can be approximated as (Crosbie and Bohn, 2003; McNeil et al., 2005)

$$\text{Distance to Default} = \frac{[\text{Market value of assets}] - [\text{Default point}]}{[\text{Market value of assets}].[\text{Asset volatility}]}, \tag{10}$$

where the asset volatility is the standard deviation of the annual percentage change in the asset value.

The KMV model premises that DD is a sufficient statistic to arrive at a rank ordering of default risk, where the numerator in (8) expresses the firm's financial leverage or *financial risk*, while the denominator reflects its *business risk*.

Following the definition of default in project finance given in (2), Distance to Default for infrastructure project finance loans at time t can be defined as

$$DD_t = \frac{CFADS_t - \mathrm{DS}_t^{\mathrm{BC}}}{\sigma_{CFADS_t} CFADS_t} \tag{11}$$

Using the definition of $DSCR_t$ in (1), the above expression can be written as:

$$DD_t = \frac{1}{\sigma_{CFADS_t}}(1 - \frac{1}{DSCR_t}) \tag{12}$$

The above can be re-written as a sole function of $DSCR_t$ by expressing the volatility of $CFADS_t$ as a function of that of $DSCR_t$ (as shown in Appendix A.1)

$$DD_t = \frac{1}{\sigma_{DSCR_t}} \frac{\mathrm{DS}_{t-1}^{\mathrm{BC}}}{\mathrm{DS}_t^{\mathrm{BC}}}(1 - \frac{1}{DSCR_t}) \tag{13}$$

where σ_{DSCR_t} is the standard deviation of the annual percentage change in the $DSCR$ value.

3.2.2. Risk Neutral Probability of Default

In the Merton model, the mapping between risk neutral and physical probabilities of default is given by (Kealhofer, 2003)

$$q(t,T) = N\left(N^{-1}[p(t,T)] + \lambda_T\right), \tag{14}$$

where $q(t,T)$ is the risk neutral cumulative probabilities of default between time t and T, and $\lambda_T = \frac{\mu - r}{\sigma}\sqrt{T-t}$ is the required Sharpe ratio over this horizon.

The corresponding risk neutral distribution for DSCR can be written as (Wang, 2002)

$$F^*(\text{DSCR}_T) = N\left(N^{-1}[F(\text{DSCR}_T)] + \lambda_T\right), \tag{15}$$

where $F(\text{DSCR}_T)$ and $F^*(\text{DSCR}_T)$ are the physical and risk-neutral distributions of DSCR_T.

If the physical distribution ($F(x)$) is normal ($X \sim N(\mu,\sigma)$), or lognormal ($ln(X) \sim N(\mu,\sigma)$), then the risk neutral distribution ($F^*(x)$) follows the same distribution (normal or lognormal) with a shifted mean $\mu - \lambda\sigma$. Hence, the risk neutral distribution of the DSCR would the same as the physical distribution of the DSCR but with a shifted mean.

3.2.3. Decomposition of Risk Into Traded and Non-Traded Components

In order to determine the required Sharpe ratio for project finance loans, we can decompose the underlying CFADS process into a component that is spanned by traded securities, and a component that is not (Froot and Stein, 1998). That is, we write the current period's CFADS as

$$CFADS_{t-1} = CFADS^T_{t-1} + CFADS^N_{t-1},$$

where $CFADS^T_{t-1}$ represents the component of CFADS generated by the replicating portfolio, and $CFADS^N_{t-1}$ represents the components of the CFADS not generated by the replicating portfolio. Then, we can write the mean return on CFADS as (see Appendix A.2)

$$\mu = w^T_{t-1}\frac{\sigma^T}{\sigma}\lambda^T + w^N_{t-1}\frac{\sigma^N}{\sigma}\lambda^N,$$

where we have defined $w^{T(N)} = \frac{CFADS^{T(N)}_{t-1}}{CFADS_{t-1}}$, and $\lambda^{T(N)} = \frac{\mu^{T(N)} - r}{\sigma^{T(N)}}$.

This separation of risks serves two main purposes:

1. The required prices for hedgable risks can be set equal to the premium earned by the traded portfolio, to prevent arbitrage.

2. The required prices for unhedgable risks would lie in an approximate arbitrage band. Within this band, the prices can be modelled (see Froot and Stein, 1998) and calibrated to the observed PF debt prices, and hence the valuation model can be used to learn about the impact of different regulatory requirements, liability structures, and other factors that may affect investors' risk preferences.

The decomposition of CFADS into traded and untraded components is an empirical task, and can only be done once sufficient data is available to estimate the correlations between CFADS and cash flows on traded securities. Thus, in the model implementation detailed in section 4, we assume that the CFADS process is completely uncorrelated with traded securities, and hence the w^T is zero.

3.2.4. Choice of Bounds on Required Risk Premium

Before proceeding with the valuation model, we discuss our choice of bounds of the required Sharpe ratio λ. We argue that the investors' Sharpe ratios would lie in a band between 0 and 2.

Indeed, annualised Sharpe ratios for market indices typically fall below 1.0, and the largest Sharpe ratios are often exhibited by hedge funds. Even for high performing hedge funds, the only instances where the Sharpe ratio may exceed 2.0 are when the returns are not normally distributed (Kat and Brooks, 2001). Non-normal distributions exhibit higher moment risks, such as negative skewness, high kurtosis, and the Sharpe ratio (which only takes into account the first two moments) can underestimate the riskiness of such investments.[6]

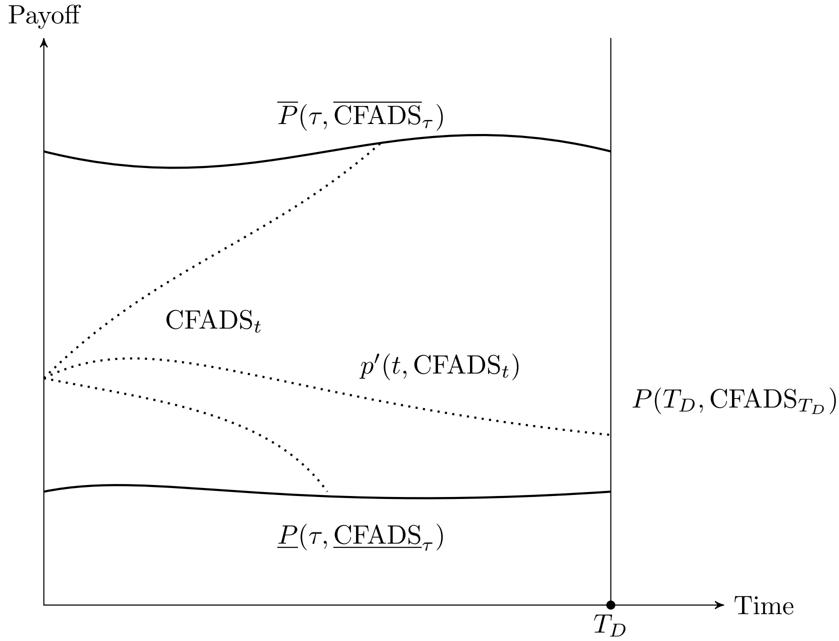
Since we assume normal distribution for the underlying risk in our examples,[7] we argue that if PF loans offered Sharpe ratios above this upper limit of 2, they would become too attractive, and such loans would soon disappear from the market. Therefore, in equilibrium, the Sharpe ratios for PF loans would lie between 0 and 2.

Theoretical justification for bounds on risk/reward ratios is discussed in Cochrane and Saa-

[6] For example, the Long-Term Capital Management (LTCM) exhibited a Sharpe ratio of 4.35 before its demise in 1998 (Lux, 2002). However, as is now well known, the hedge fund was exposed to some extreme risks, and the return distribution was highly non normal.

[7] For non-normal distributions, the bounds can be specified using other risk reward ratios, such as the gain-loss ratio introduced by (Bernardo and Ledoit, 2000).

Figure 1: Black-Cox decomposition at one point in time.



Requejo (2000); Bernardo and Ledoit (2000). Cochrane and Saa-Requejo (2000) show that even with high levels of risk aversion and volatility in future levels of consumption, Sharpe ratios do not exceed 1.72. Hence, our choice of an upper limit of 2.0 seems justified from both an applied and a theoretical perspective.

3.3. Black Cox Decomposition

Once cash flow dynamics are known under the risk-neutral measure, we must take into account the frequent restructurings of project finance loans once an credit event has occurred. In this section, we adapt the Black-Cox decomposition to the case of project finance.

The Black-Cox decomposition (Black and Cox, 1976) was devised to value corporate securities when firms can reorganise. However, the original model assumes that the reorganisations happen when the total value of the firm reaches a lower or an upper boundary, whereas reorganisations in project finance are determined not by the total value of the SPE, but by the CFADS at each point in time. Therefore, we modify the Black-Cox decomposition to take into account this difference. We define 4 payout functions for PF loans, as illustrated by figure 1 :

1. $P(T_D, \text{CFADS}_{T_D})$: final payment at the maturity of the contract. (We use T_D to refer to

the maturity of the debt contract, which may be different from the maturity of the project denoted earlier by T.)

2. $\underline{P}(\tau, \text{CFADS}_\tau)$: the value of the corporate security if the CFADS reaches the lower boundary at time τ.
3. $\overline{P}(\tau, \text{CFADS}_\tau)$: the value of the corporate security if the CFADS reaches the upper boundary at time τ.
4. $p'(t, \text{CFADS}_t)$: the payments made by the debt security until the maturity or reorganisation.

The total value of a PF loan is the expected present value of the sum of these 4 payout functions under the risk-neutral measure, discounted at the risk-free rate. Next, we model the outcome of debt restructuring in project finance.

3.4. Debt Restructuring

Identifying DSCR dynamics leads to a $CFADS_t$ model conditional on the base case debt schedule. However, as discussed in section 2.2, the base case debt schedule can change upon the reorganisation that follows a credit event in project finance. The resulting change in the debt schedule also changes the expected DSCR profile post-reorganisation. To capture this change in the DSCR profile, we need to model the change in debt schedule.

To model reorganisations upon default, we assume that the equity holders honour their debt obligations as long as their is sufficient free cash flow (CFADS) available to make the scheduled debt payment.[8]

3.4.1. Restructuring upon a technical default

Covenant breach or technical default gives debt holders the right to step-in, impose certain management decisions, and require the restructuring of the outstanding debt. In practice, equity investors may also be required to inject more capital in the project company, but we ignore this possibility and assume that debt is only paid with the free cash flows of the SPV.

[8]Alternatively, we could have assumed that the equity holders could pursue strategic debt service. However, we ignore this possibility because in project finance the funds for non-operating expenses may be held under joint control of lender's agent and the project company (see section 13.5 in Yescombe, 2002). Therefore, the equity holders are not likely to have enough control on free cash flow to pursue strategic debt service.

In a situation of technical default, **lenders can aim to maximise the value of the restructured debt service relative to the original outstanding debt amount**,[9] but not more.

Indeed, the project company does not go into bankruptcy and equity holders continue the construction and/or operation of the project (see Gatti, 2013, section 7.2.3.11.2 on negative covenants). They do not exercise their limited liability option and retain significant control rights as owners of the project company.

We further assume that debt holders will have to incur some *restructuring costs* to have the debt reschedules. Therefore, they only choose to reschedule the outstanding debt if they can impose a new debt schedule such that the *market value* of the new debt, net of restructuring costs, is higher than the market value of the existing debt.

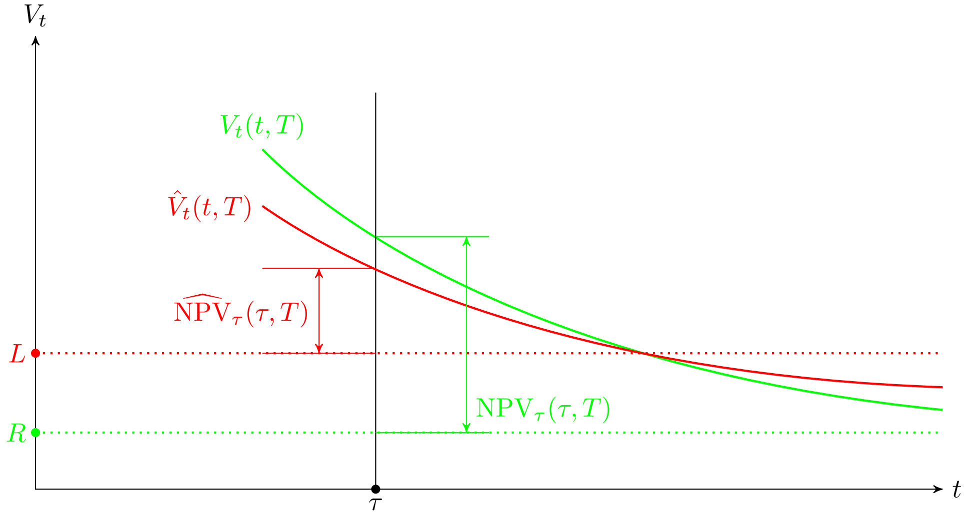
Thus, restructuring PF debt upon a technical default involves the following steps.

1. Determine the *outstanding debt value*: the present value of the existing debt schedule discounted at the original IRR of the loan;
2. Determine the *market value* of the existing debt schedule i.e. the risk-adjusted value of debt discounted at the appropriate rate, which is likely to be different from the original IRR. We propose determine the market value of the debt using a risk neutral valuation model as discussed in section 3.2;
3. Pick a new debt schedule such that its value when discounted at the original IRR of the loan is the same as the original outstanding debt value;
4. Determine the market value of this new debt schedule;
5. If the market value of the new debt schedule, net of rescheduling costs, exceeds the market value of the original debt schedule, the new debt schedule is preferred;
6. These steps can be repeated until a debt schedule has been found that maximises the market value of the restructured debt, for example by minimising credit risk and extending the debt service in the "tail" of the loan.

Thus, a situation of technical default gives lenders **control rights that allow them to maximise their expected recovery rate**. Technical defaults are the most frequent type of credit

[9]The outstanding debt at any point is simply the amortised value of the debt, which can be obtained by discounting the remaining scheduled debt payments by discounting them at the internal rate of return.

Figure 2: Renegotiation and liquidation values at the time of default.



event in project finance for the obvious reason that the CFADS is more likely to reach some threshold set before a hard default can occur, than to actually lead to a default of payment.

3.4.2. Restructuring upon a hard default

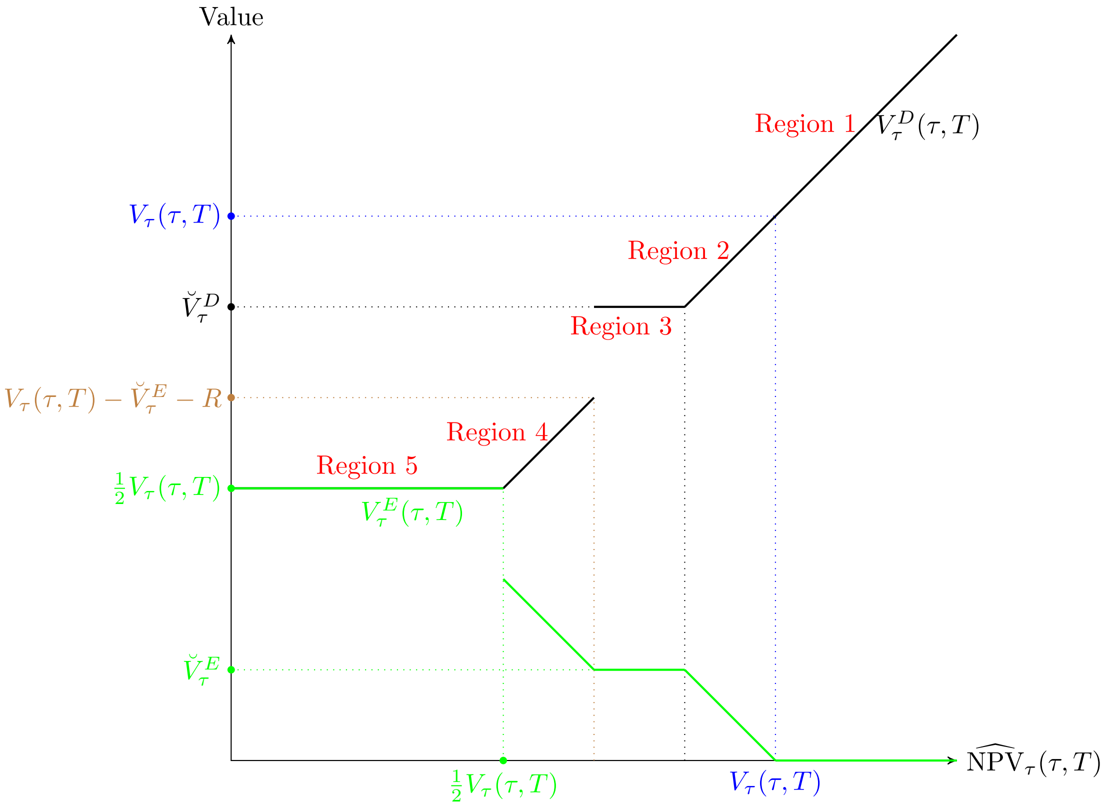
Hard defaults create a more complex set of outcomes. We treat a hard default as an event upon which the existing contract between debt and equity holders is impaired, and equity holders loose the control rights of the SPE, which is the result of their original *share pledge*. However, because equity holders can now exercise their limited liability option, *depending on the costs to lenders implied by an actual take-over of the SPE*, the original equity holders have not lost all bargaining power.

After a hard default, lenders have the control of the SPE and they can aim to maximise the value of these control rights. Their preferred course of action may or may not involve the original equity owners. Next, we describe the conditions under which renegotiations can take place upon a hard default and their possible outcomes in the debt renegotiation model.

We assume that the following outcomes are possible upon a hard default:

- Bankruptcy or sale of the company: Debt and equity holders either file for bankruptcy or sell the SPE and receive a share according to their seniority;

Figure 3: The outcome of renegotiation as a function of liquidation value of the SPE at the time of default. $\widehat{\text{NPV}}_\tau(\tau,T)$ denotes the liquidation value of the SPV at the time of default, and $V_\tau(\tau,T)$ denotes the value of SPV under existing ownership. $V_\tau^D(\tau,T)$, $V_\tau^E(\tau,T)$, and $V_\tau(\tau,T)$ denote the value of debt, equity under renegotiated debt schedule; and $\check{V}_\tau^D$ and $\check{V}_\tau^E$ denote the value of debt and equity under existing debt schedule, respectively. L and R denote liquidation and renegotiation costs, respectively. The black and green lines show the values of debt, and equity upon renegotiation.



- Takeover: debt holders enter into a new contract with a new set of owners;
- Sale of the loan: debt holders sell their loan in the secondary market;
- Renegotiation: debt and equity holders enter into a new contract.

Since, debt owners are the sole owner of the SPE upon default and we assume that they will choose the course of action that maximises their value since they are the sole decision makers. In the first three cases, debt and equity holders do not enter into a new contract. We refer jointly to these outcomes as the *liquidation scenario*. In contrast, in the fourth case, which we label the *renegotiation scenario*, debt holders enter into a new contract with existing equity holders. Empirical studies on project finance suggest that the most common scenario upon a hard default is a 'work out' (Moody's, 2014), which is equivalent to our *renegotiation scenario*. Next, we discuss the conditions under which renegotiations can take place and their possible outcomes.

We denote the value of debt and equity upon liquidation as their *liquidation value*. Here, debt renegotiation only occurs if the following conditions are satisfied:

(C_1) Both debt and equity holders can gain at least as much from renegotiation as they can from liquidation, i.e.

$$V_\tau^i(RN) \geq V_\tau^i(LQ), \text{ for } i = D \text{ and } E$$

(C_2) At least one of the stake holders can get more than what they do under the existing contract, i.e.

$$V_\tau^i(RN) > \check{V}_\tau^i, \text{ for } i = D \text{ or } E$$

(C_3) Debt holders never obtain less than the equity holders, as they are the owners, i.e.

$$V_\tau^D(RN) > V_\tau^E(RN)$$

where D stands for debt, E for equity, RN for renegotiation, and LQ for liquidation. Thus, $V_\tau^i(RN)$ denotes the value of i^{th} stakeholder ($i \in [D, E]$) upon renegotiation, and $\check{V}_\tau^i$ denotes the value of i^{th} stakeholder under no change in existing debt schedule.

If the first condition does not hold, at least one of the parties will have no incentive to participate in renegotiation, and hence renegotiation does not occur. If the second condition does not hold, no party has have an incentive to renegotiate, and renegotiation does not occur. The third condition simply postulates that debt holders, being the effective owners of the SPE upon default, should be able to secure at least half of the value of the SPE in the renegotiation.

Consistent with the non-recourse nature of project company, and the relationship-specific nature of its assets, we assume that the liquidation scenario corresponds to debt holders taking over the project company. That is, if renegotiation does not take place, debt holders is take over the SPE, and run it either by themselves or seek new equity owners. Hence the liquidation value of the debt is the net present value of the cash flows under debt holders' ownership net of any costs associated with taking over the SPE. Since the equity holders get no share in the new company in this case, the value of equity upon liquidation is zero[10]. The *liquidation values* of debt and equity owners are the lower bound of the *renegotiation values*, and provide an intuitive reason why renegotiation can happen.

The liquidation values of debt and equity can be written as

$$V_\tau^D(LQ) = \max\left(\hat{V}_\tau - L_\tau, \text{Cash}_\tau\right), \tag{16}$$

$$V_\tau^E(LQ) = 0, \tag{17}$$

where L_τ represents liquidation costs at time τ. We assume that the liquidation costs are constant in time, and renegotiation costs can be either 0 or R, and that debt and equity holders have identical risk preferences, and expectations about future cash flows.

As shown in the figure 2, if the liquidation value of the SPE (present value of the cash flows under alternative ownership net of liquidation costs) is lower than its renegotiation value (present value of the cash flows under existing ownership net of renegotiation costs), then both debt and equity holders should be better off renegotiating the contract, rather than liquidating the firm. In other words, if the renegotiation value of the SPE is sufficiently high compared to its liquidation value of, debt holders can obtain more than their liquidation value, even after sharing a fraction of the SPE with equity owners, who get nothing in the liquidation case.

Thus, both the feasibility of renegotiation and its outcome are influenced primarily by the liquidation value of the SPE. In the extreme case where the SPE is worth nothing upon liquidation,

[10]Here, we assume that equity owners' opportunity cost of owning the project is zero. In reality, equity holders would have to commit their time and exert effort in running the firm. Hence, their liquidation value would be the value of this time and effort spent on running an alternative comparable project. Incorporating this non-zero opportunity cost could be one of the avenues for future extensions of this model.

debt holders have no choice but to renegotiate with existing equity holders.

Under this set of assumptions, the value of debt is bounded from below by $max\left(\hat{V}_\tau - L), \frac{1}{2}V_\tau, \text{Cash}_\tau\right)$[11] the following scenarios can be envisaged:

1. $\hat{V}_\tau - L > V_\tau$: In this case, the liquidation value of the firm is greater than the existing value of the firm, and debt holders are better off liquidating the firm. Hence, there will be no renegotiation in this case.
2. $max\left(\frac{1}{2}V_\tau, \text{Cash}_\tau\right) < \hat{V}_\tau - L < V_\tau$: In this case, the liquidation value is higher than what debt holders could get by equally sharing the value of the existing firm with the equity owners. This scenario can be further sub-divided into the following scenarios
   (a) $\hat{V}_\tau - L > \check{V}_\tau^D$: In this case, debt holders seek to benefit from default, and they will force equity holders to increase the value of debt to the liquidation value of the SPV. Hence the debt and equity values will be

$$V_\tau^D = \hat{V}_\tau - L, \tag{18}$$

$$V_\tau^E = V_\tau - (\hat{V}_\tau - L). \tag{19}$$

   Renegotiation costs will not be incurred in this case, because if equity holders do try to impose renegotiation costs on debt holders, debt holders would simply liquidate the firm. Hence, equity holders would simply let the debt holders increase their debt value, and renegotiation will be costless.

   (b) $V_\tau - (\hat{V}_\tau - L) - R > \check{V}_\tau^E$: In this case equity holders can benefit from default, and they would force the debt holders to offer concessions and reduce the value of the debt to the liquidation value of the SPV. Hence the values of debt and equity will be

$$V_\tau^D = \hat{V}_\tau - L, \tag{20}$$

$$V_\tau^E = V_\tau - (\hat{V}_\tau - L) - R. \tag{21}$$

   In this case, the equity holders would have to incur renegotiation costs, because the debt holders would not lower their value unless equity holders force them to do so.

   (c) Neither one of the above two conditions holds: In that case neither party stands to

[11]This is consistent with the commonly used formulas for the outcome of noncooperative bargaining with outside options (Hart and Moore, 1994; Osborne and Rubinstein, 1990).

benefit from default, and they simply continue with the existing debt schedule

$$V_\tau^D = \breve{V}_\tau^D, \tag{22}$$

$$V_\tau^E = \breve{V}_\tau^E. \tag{23}$$

3. $\frac{1}{2}V_\tau > max\left(\hat{V}_\tau - L, \mathrm{Cash}_\tau\right)$: In this case, the value of liquidation option is so low that the debt holders are better off equally sharing the existing value of the firm with equity holders. Hence the values of debt and equity holders are given by

$$V_\tau^D = \frac{1}{2}V_\tau, \tag{24}$$

$$V_\tau^E = \frac{1}{2}V_\tau. \tag{25}$$

4. $\mathrm{Cash}_\tau > max\left(\hat{V}_\tau - L, \frac{1}{2}V_\tau\right)$: In this case, the value of cash available in the current period is greater than the value of SPV as a going concern. Hence, the debt holders simply take the cash available at hand and the SPV ceases operations.[12] Hence the debt and equity values are

$$V_\tau^D = \mathrm{Cash}_\tau, \tag{26}$$

$$V_\tau^E = 0. \tag{27}$$

Figure 3 graphically illustrates the outcome of renegotiation as a function of the liquidation value, assuming $\frac{1}{2}V_\tau > \mathrm{Cash}_\tau$.

The new debt schedule upon a hard default is computed such that the present value of the debt schedule is equal to the value of debt as determined by the renegotiation model. In principle, many different debt schedules can be determined that yield the same present value, and hence the debt holders would be indifferent under these different debt schedules. For simplicity, we assume that the new debt schedule is determined such that the loan has a constant DSCR, and the maturity of the loan coincides with the maturity of the project.

3.4.3. Refinancing

To model the outcome of reorganisations at the upper boundary or refinancings, we make a few simplifying assumptions. Firstly, we ignore the effects of market conditions: the level of

[12] In theory, equity holders could offer debt holders to retain the available cash, and continue to run the firm. However, since debt holders are the owners and they have no incentive to renegotiate in this case, we assume that the company ceases to exist.

interest rates, demand for PF debt etc, and assume that the refinancing does happen as soon as the CFADS hits a predetermined boundary. In other words, we assume that as soon as the $CFADS_t$ crosses a certain threshold, the project's level of riskiness decreases sufficiently to justify a reduction in the cost of debt, irrespective of market conditions. Secondly, we assume that upon refinancing, the amount of debt outstanding is paid in full along with any costs or penalties imposed by the debt covenants.

In the Black-Cox decomposition discussed above, the value of debt at the upper reorganisation boundary is given by

$$\overline{P}(\tau) = (1+c)\left[\sum_{i=\tau}^{T_D} e^{-rate(i-\tau)}\mathrm{DS}_i\right], \tag{28}$$

where c is the refinancing costs, $rate$ is the original IRR of the loan, and DS_i^{BC} is the scheduled debt payment at time i.

4. Implementation

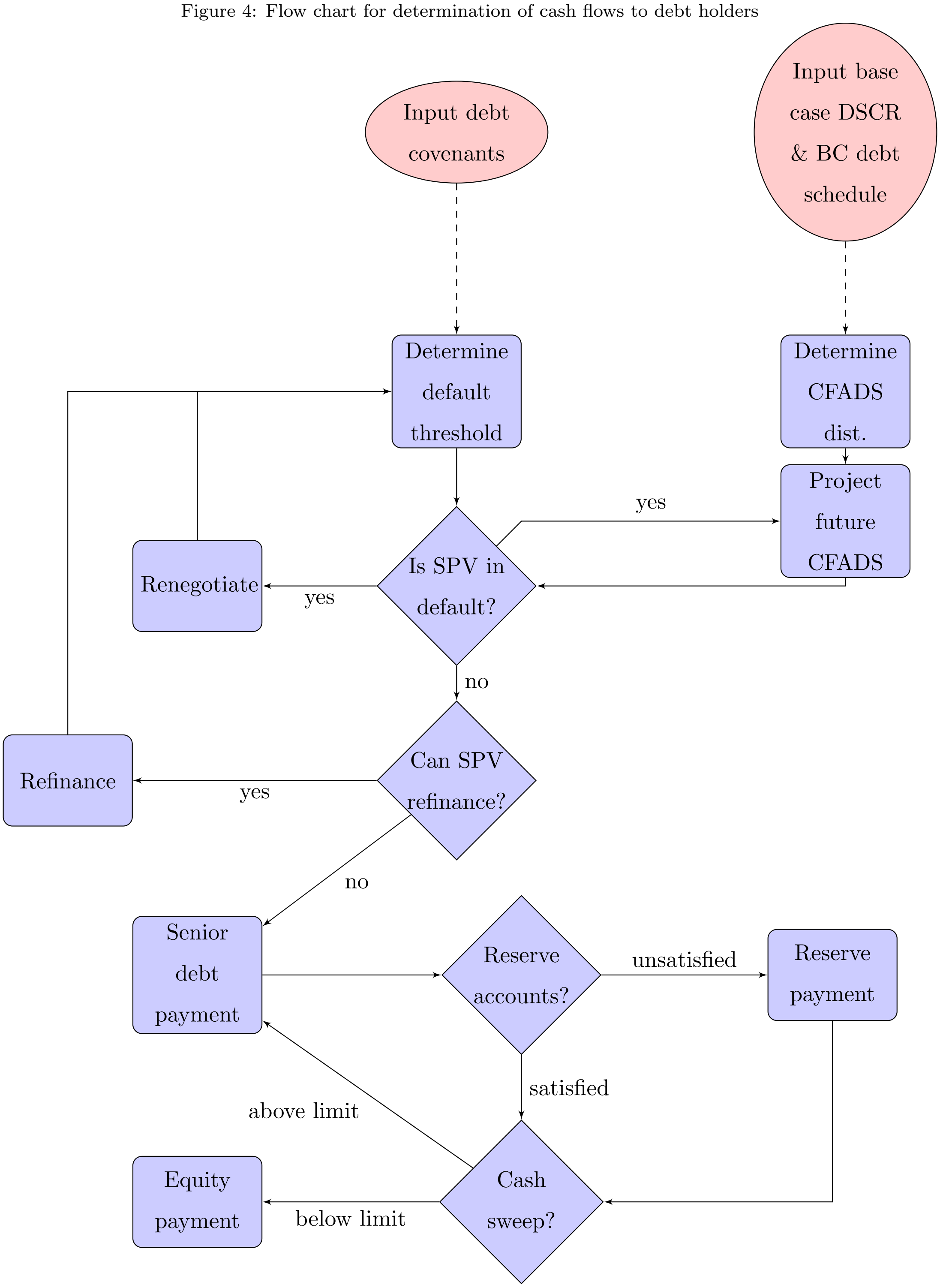
In this section, we implement our model for two generic types of infrastructure projects: a *merchant* infrastructure project and a *contracted* infrastructure project. We first describe the algorithm implementing the model described above and then detail our choice of inputs.

4.1. Algorithm

In this section, we provide an algorithm for the numerical implementation of the theoretical model, as illustrated by figure 4. The main steps in implementing the framework are

1. Obtain the base case debt schedule;
2. Obtain the base case DSCR profile, and select a model for DSCR distribution;
3. Determine the CFADS distribution using the DSCR distribution and the base case debt schedule;
4. Risk neutralise the distribution of the CFADS: Select a required Sharpe ratio, and shift the original DSCR (or CFADS) distribution accordingly;
5. Obtain debt covenants: Debt covenants may contain reserve accounts, cash sweeps and clawback provisions etc. and include the technical default threshold: the threshold below which lenders have the right to step in and reschedule the debt;

Figure 4: Flow chart for determination of cash flows to debt holders



6. Project CFADS paths for future periods using the distribution obtained above;
7. Determine if the SPV is able to refinance: for each projected CFADS path, determine if the SPV has transitioned into a sufficiently low risk environment where it can refinance its debt;
    - If refinancing is possible, i.e. if the projected CFADS exceeds the refinancing threshold, determine the new debt covenants (debt service schedule, reserve account requirements etc.). All debt covenants need not change, and the only change may be in the debt service schedule and the default threshold;
8. Determine if the SPV is in default: Compare the projected CFADS for each period with the default threshold;
    - If the SPV is in technical default, debt is rescheduled if a new debt schedule can be found that exceeds the existing debt schedule's market value;
    - If the SPV is in hard default, the new debt schedule is determined based on the outcome of renegotiation model;
9. Construct the cash flow waterfall with existing debt covenants: making payments according to the priorities established in the debt covenants, which would include payments to debt holders, reserve accounts, and equity holders;
10. Once cash flows to the debt holders have been projected, the present value of these cash flows is calculated under the risk-neutral probability measure using risk-free discount rates.

4.2. Inputs

Table 1 provides our characterisation of our two generic project structures. Both projects last for 25 year. The merchant project has a 5 year construction period, is financed with 75% leverage[13], the loan is repaid between year 6 and 19, hence a tail of 6 years.

The contracted infrastructure project has a 3 year construction period, is financed with 90% leverage, and repays the loan between years 4 and 23, leaving a tail of 2 years. Total initial debt is normalised to 1,000.

[13] We define leverage as the ratio of the market value of the loan to the market value of the SPV at financial close. Hence, the leverage is sensitive to the risk preferences of the investor. Different choices of risk preferences (Sharpe ratio) may lead to different values of debt and SPV, and hence the leverage may change. The leverage given in the table is for a benchmark investor with a Sharpe ratio of 1.

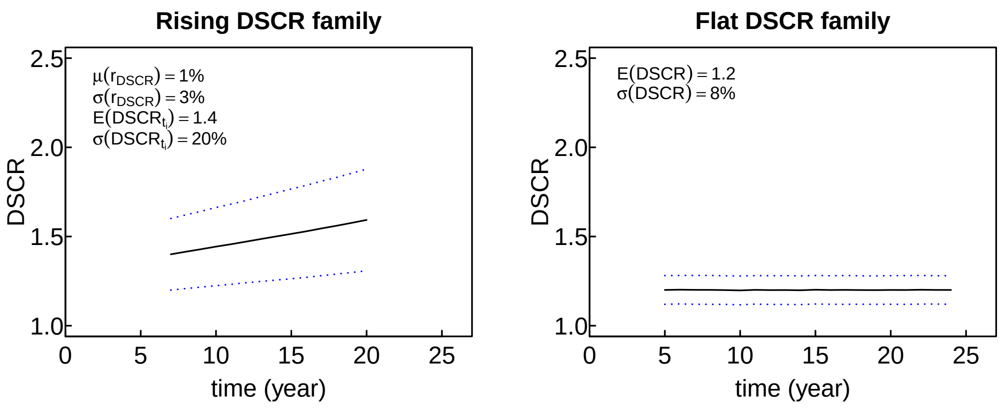
Table 1: Merchant and contracted infrastructure projects

Project type	Construction period	Tail length	DSCR profile	Project maturity	First payment	Final payment	Base case IRR
Merchant	5 year	6 year	Rising	25	Year 6	Year 19	4%
Contracted	3 year	2 year	Flat	25	Year 4	Year 23	3.5%

Table 2: DSCR models for the two DSCR families.

DSCR profile	DSCR distribution	Mean Return	Volatility of returns	Initial expected DSCR	Volatility of initial DSCR
Rising	Lognormal	1%	3%	1.4	20%
Flat	Normal	NA	NA	1.2	8%

Figure 5: DSCR models for merchant and contracted infrastructure projects.



At this stage, before empirical observations can be made, we model the DSCR for the merchant project using a lognormal distribution with a constant mean return (increase) of 1%, a constant volatility of returns of 3%, an initial DSCR of 1.4, and 20% volatility of the initial DSCR. That is, ex-ante, the DSCR of the project is expected to be 1.4 with a standard deviation of 20% immediately after construction, and is then expected to rise lognormally with 1% mean return and 3% volatility in returns.

The DSCR for the contracted project is modelled using a normal distribution with a mean DSCR of 1.2, and a volatility of 8%. Hence, *ex ante*, the DSCR for the contracted project is expected to be normally distributed around 1.2 with a standard deviation of 8% for the entire life of the loan.

We list the model parameters for the two DSCR distributions in table 2. Figure 5 shows the projected DSCRs for both families of DSCR dynamics.

The base case DSCR is available only until the original maturity of the loan. However, in order to take into account the value of tail, one needs to project CFADS in the tail. For that matter, we assume that the CFADS distribution does not change upon loan's maturity. In the two examples discussed above, the CFADS follows the same distribution as the DSCR, as the debt payments are constant in time. Thus, we project the CFADS in the tail using the same distribution that was used during the life of the loan. This is a simplifying assumption, as project finance debt service is often 'sculpted' but this is an approximation across a basket of loans.

5. Results

In this section we compute risk and return measures for two projects under the following assumptions

- Equity dividends are locked up if DSCR falls below 1.10, and technical default is triggered if DSCR falls below 1.05;
- Liquidation costs are 60% of initial value of debt;
- Renegotiation costs are one half of liquidation costs;
- Restructuring costs are one third of liquidation costs;

- We ignore the Region 3 in figure 3, and assume debt value is equal to the liquidation in this region (this is to simplify numerical computation and in unlikely to have any significant influence on the risk profile;
- We ignore refinancing, i.e. we assume that upper boundary in the Black Cox decomposition is at infinity.

5.1. Risk Measures

In this section, we compare the risk profile[14] of the two DSCR families. This includes a comparison of debt payments, probability of default, per period losses, and value-at-risk, and conditional value-at- risk (expected shortfall).

We find relatively low levels of credit risk (compared to the current treatment of infrastructure debt in Solvency-II for instance) and we note that the default and recovery dynamics predicted by the model are in line with those reported by ratings agencies (see for instance Moody's, 2014).

5.1.1. Ex-ante Risk Profile

First, we show the *ex ante* risk profile for each DSCR family. Figure 6 compares the CFADS, and mean debt payments for the two families. We see that on average, in both cases, mean debt payments gradually fall below the base case debt schedule, but exceed the base case debt schedule in the tail, hence reducing losses incurred in the earlier periods.

Figure 7 shows the probability of default (PD) for the two families. The PD decreases rapidly for the rising DSCR family, while it stays nearly constant for the flat DSCR family. This is because the rising DSCR for the merchant projects makes it unlikely for the project to default if it survives first few years post construction. While the flat DSCR for contracted projects imply that the loan is equally likely to default throughout the life of the loan.

Figure 8 compares the loss profile (loss per period, VaR, cVaR) for the two families. In the case of the flat DSCR family, mean loss, VaR, and cVaR all rise towards the maturity of the debt. In the case of rising DSCR family, while the mean losses do rise, VaR and cVaR stay constant near the maturity of the debt.

[14]Definitions of risk measures are given in Appendix B.

Figure 6: Comparison of mean debt payments, and CFADS.

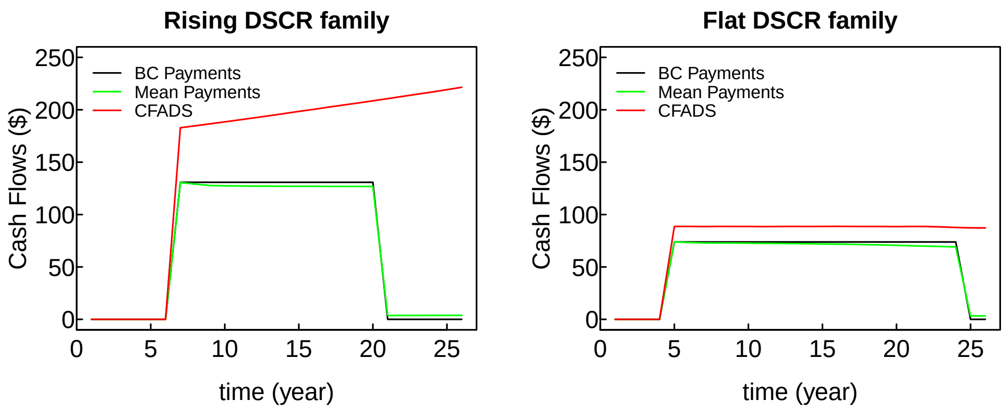


Figure 7: Comparison of probabilities of default and death for the two DSCR families. The black line includes both technical and hard defaults. The Green line only includes hard defaults for projects that have not defaulted before, and the red line shows the probability of project company going bankrupt.

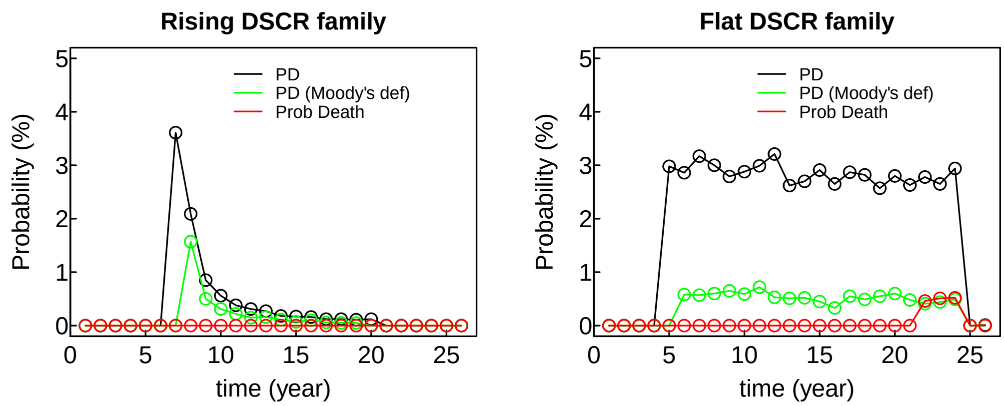
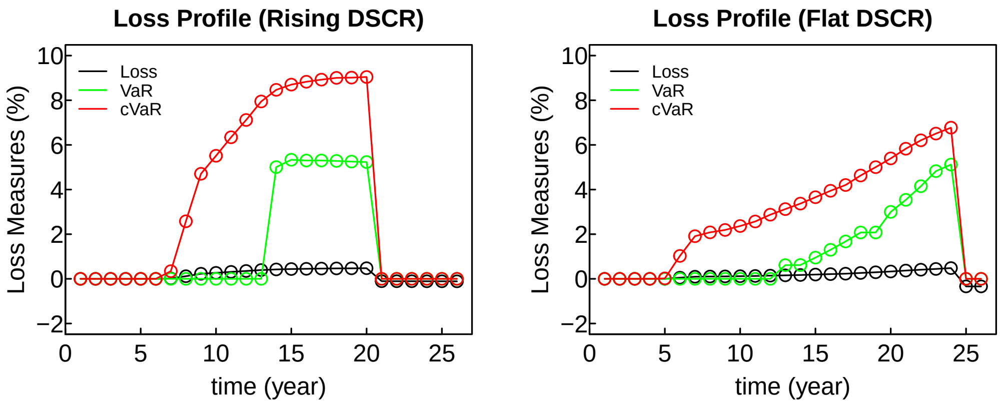


Figure 8: Comparison of loss, VaR, and cVaR for the two familes.

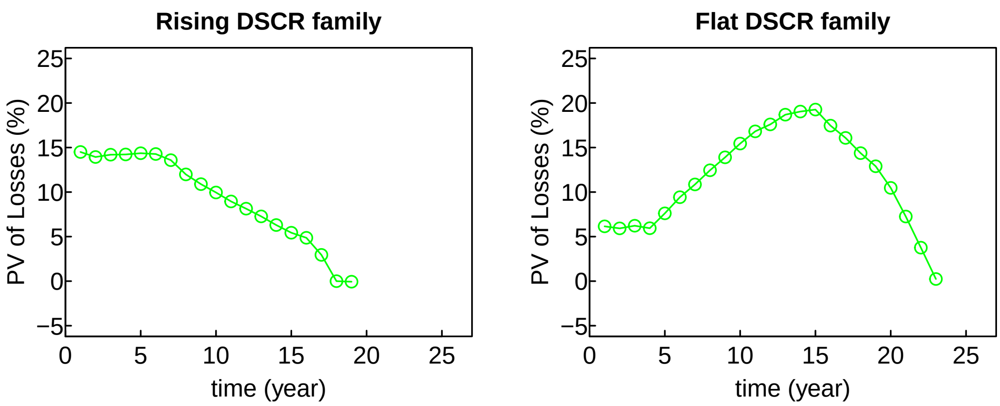


The rising trend in per period losses (both mean and extreme losses) can be explained by increasing cumulative probability of default. As more defaults occur over time, debt holders get a hair cut, and post-default mean debt payments decrease. Therefore, mean debt payments near the maturity of the loan reflect the accumulated effect of hair cuts due to all the defaults in prior periods. This is why mean debt payments are lower near debt maturity (as seen in figure 6), and mean losses are higher, even for the rising DSCR family for which the marginal default probability near maturity is close to zero.

The difference in the VaR and cVaR trends between the two families stems from the different tail values. In the case of flat DSCR family, the lower tail value and relatively higher leverage near the tail of the loan increases the severity of defaults compared to the defaults in the earlier periods. This is because the tail is very short and the mean CFADS stays constant. Therefore, if a default occurs near the maturity of debt, there may not be enough cash in the tail to cover the losses. That is, the defaults near the maturity of debt can be more costly than the defaults during the earlier periods. In the case of rising DSCR family, the tail is relatively longer and CFADS is rising, hence there is a lot more cash available in the tail of the project. Therefore, the severity of losses is not so much affected by the timing of defaults.

We see the effect of different tail values further in the distribution of deaths. We use 'death' to denote an outcome where the project company ceases to be a going concern upon default. This happens when cash available upon default, including the cash in the reserve accounts, exceeds the value of SPV in operation. Thus, debt holders are better off taking the available cash, and

Figure 9: Present value of expected losses as a percentage of the value of debt.



letting the SPV go bankrupt. In the case of the rising DSCR family, we do not see any deaths due to higher tail value that makes SPE more valuable as a going concern. While for the flat DSCR family, the lower value of the tail makes it more likely for a hard default to lead to death near the maturity of the project, as there is not a lot of cash left in the remaining periods. Note however that the probability of death remains very low at around 0.5%.

5.1.2. Time Evolution of Risk

Expected Losses

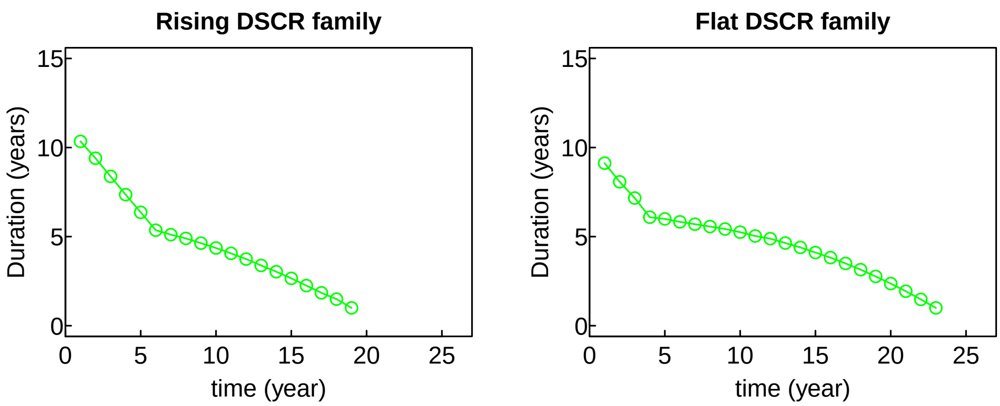
Next, we show the time evolution of expected losses and duration of the loan, assuming that the base case debt payments are realised in every period. That is, we move forward in time, collect base case debt payments in every period, and compute the expected present value of losses and duration in each period.

Figure 9 shows the present value of expected losses as a percentage of existing value of debt and in absolute terms, respectively. For the rising DSCR family, expected losses decrease in time, while, in the case of flat DSCR family, the expected losses first increase, and then decrease.

The evolution of expected losses is driven by

1. the time resolution of uncertainty as we move forward in time, which decreases expected losses;
2. and an increase in losses as we get closer to the period of higher losses.

Figure 10: Time evolution of duration.



In the case of rising DSCR family, the first effect dominates, and the expected losses (recovery rates) go down (up). For the flat DSCR family, for which the losses are more concentrated near the maturity of the loan, the second effect dominates in the beginning, leading to an increase in expected losses. However, as we move past this region, the first effect begins to dominate, and the expected losses (recovery rates) go down (up).

Duration

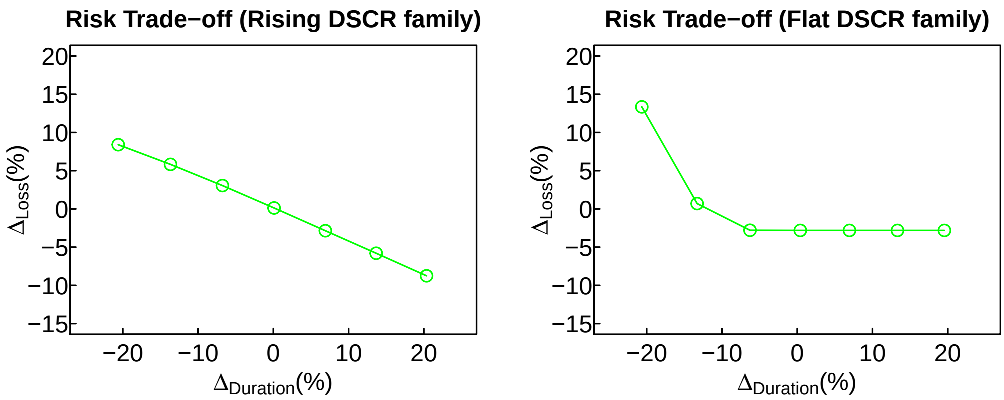
Figure 10 shows the time evolution of effective duration for the two families. Both families show a largely similar trend.

Figure 11 shows the relationship between losses and duration upon a hard default, when the value of debt is given by the outcome of renegotiation, but the choice of debt schedule can be in debt holders' control. That is, debt holders can choose amongst various debt schedules that have the same value at the time of default.

In this figure, we show how losses and duration are affected by the choice of debt schedule. Each point in the figure is obtained be setting a maturity for the new debt schedule, and then computing the debt payments so that the value of debt schedule is equal to the value of debt as determined by the debt renegotiation model. Debt schedules that lead to lower losses are the ones with higher duration (and longer maturity)..

While this figure shows the trade-off for one specific CFADS path, the trade-off is independent of the choice of the path. This is because in order to reduce expected losses, DSCR has to be

Figure 11: Traded-off between credit and interest rate risk. The x-axis shows the duration relative to the mean duration, and the y-axis shows the loss relative to the mean loss. The different points correspond to debt schedules with different debt maturities, chosen upon a hard default.



kept sufficiently high, which decreases the potential size of renegotiated debt payments, and as a consequence increases duration[15].

Hence, there exists a trade-off in infrastructure project finance debt between credit risk and duration risk.

5.2. Return Measures

In this section, we discuss the return measures[16] for the two DSCR families: yield, and z-spread both for a benchmark investor with a Sharpe ratio of 1, and for the investors at the two extremes of our Sharpe ratio band $[0, 2]$.

Figure 12 compares the yield and z-spread for the two families: the yield for the rising DSCR family largely stays at the same level, while the yield for the flat DSCR family increases in time before peaking and plunging towards the risk free rate near the loan's maturity.

This difference arises due to the different loss profiles. As can be seen in figure 9, the expected losses increase in early periods for the flat DSCR family, which increases its yield. Near the

[15]This argument would not hold if liquidation costs are so low that the debt holders can benefit from default. In this case a debt schedule with lower duration would also have a lower credit risk. However, we ignore this possibility, as liquidation costs are unlikely to be sufficiently low.

[16]Definitions of return measures are given in Appendix C.

Figure 12: Comparison of yield, and z-spread for the two DSCR families.

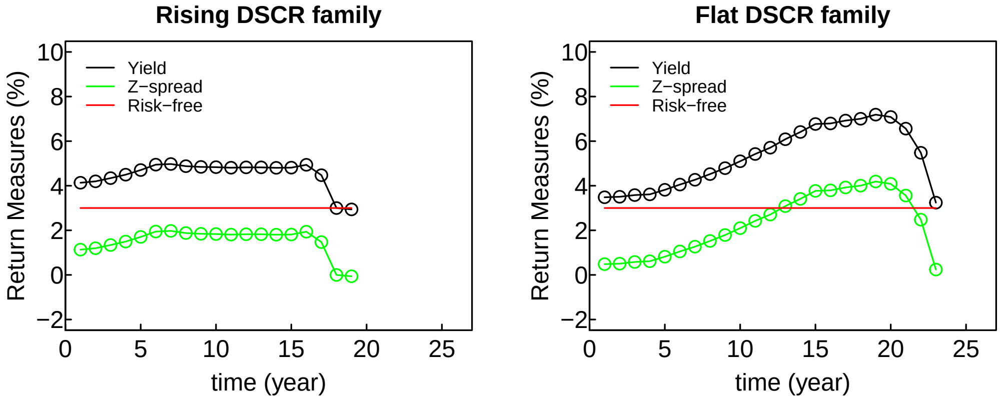
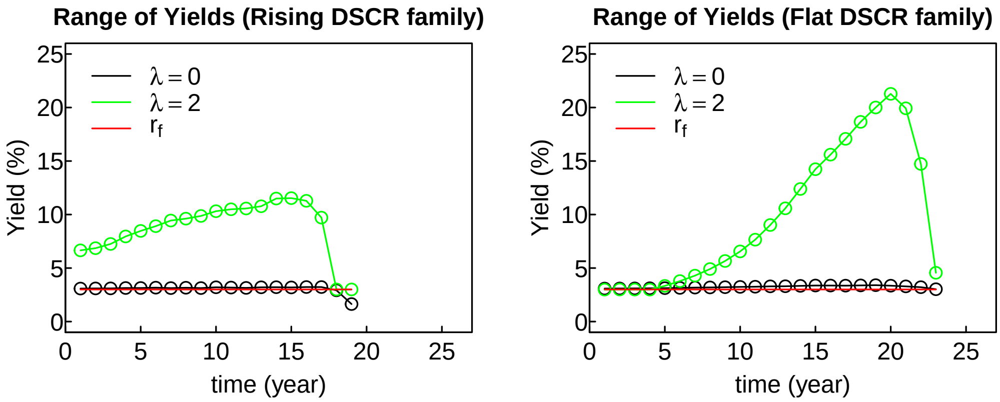


Figure 13: Range of yields for the two DSCR families.



end of loan's life, when expected losses stop increasing, the yield also stops increasing, and then converges towards the risk free rate as the value of expected losses reaches zero near the loan's maturity.

For the rising DSCR family, the expected present value of losses increase until the first payment (year 5), and decrease linearly. Hence, the yield also increases in the first few years, and then stabilises at a constant level above a risk free rate. Near the maturity of the loan, when the expected losses for the rising DSCR family go to zero, the yield also approaches the risk free rate.

Finally, in figure 13 we show the range of yields for the two extreme values of the required Sharpe ratio. While for the flat DSCR family, both yield curves remain above the risk free rate, the lower bound on the yield curve for the rising DSCR family falls below the risk free rate near the maturity of debt. This difference arises due to differences in the tail values. For the flat DSCR family, the tail value is limited and there is little scope for rescheduling the debt near the maturity.[17] Therefore the value of debt near maturity is determined simply by its scheduled debt payments, as there is no scope for rescheduling.

In the case of rising DSCR family, the tail value is sufficiently high even near maturity. As a result, debt owners can reschedule their debt upon default and get more than the promised debt payments. Therefore, the value of debt exceeds the scheduled debt payments near the debt maturity, and the yield falls below the risk free rate. This effect is more pronounced for an investor with a low level of risk aversion, as this leads to higher values of debt and a lower yield, all else being equal.

6. Conclusion

In this paper, we have designed the first methodology to compute relevant risk and return measures for long-term investors in infrastructure debt, as well as for the purpose of better calibrating prudential regulatory framework. Our framework allows the computation of expected loss, expected recovery rates, loss given default, value at risk, short fall, duration, yield, and z-spread. We have also defined a parsimonious data collection requirement for infrastructure

[17]This can also be seen from figure 7, where the probability of hard default is equal to the probability of death near the debt maturity, indicating that almost all hard defaults lead to death near the maturity of debt.

project loan valuation (summarised in annex Appendix D). Our model derives the risk-return profile from the DSCR profile, tail value, loan covenants, and liquidation and renegotiation costs and can reproduce the observed probabilities of default and recovery rates for reasonable values of input parameters.

It further suggests that risk levels are relatively low compared to those assumed in risk-based prudential framework such as the Solvency-2 regulation of insurance companies. We show that infrastructure project finance debt has a dynamic risk profile and that restructurings spread losses over the entire life of PF loans. Moreover, it suggests that recovery rates are influenced by the time variation of bargaining power that stems from project company's rate of deleveraging. Thus, rating PF loans solely on the basis of their probability of default and assumed recovery rates, averaged across projects with different DSCR profiles — which is the current practice — can lead to mis-estimating their risk profile. Finally, we show the existence of a trade-off between duration and credit risk in project finance, which is relevant to long-term investors that are attracted to such instruments for the purpose of both liability (duration) hedging and improving the risk-adjusted performance of their fixed income portfolio.

Appendix A. Proofs

Appendix A.1. Relation Between DSCR and CFADS Volatility

$$
\begin{aligned}
r_{\text{CFADS}_t} &= \frac{\text{CFADS}_t}{\text{CFADS}_{t-1}} - 1 = \frac{\text{DS}_t^{\text{BC}}}{\text{DS}_{t-1}^{\text{BC}}} \frac{\text{DSCR}_t}{\text{DSCR}_{t-1}} - 1 \\
\Rightarrow \sigma_{\text{CFADS}_t} &= \sigma\left(\frac{\text{DS}_t^{\text{BC}}}{\text{DS}_{t-1}^{\text{BC}}} \frac{\text{DSCR}_t}{\text{DSCR}_{t-1}} - 1\right) = \frac{\text{DS}_t^{\text{BC}}}{\text{DS}_{t-1}^{\text{BC}}} \sigma_{\text{DSCR}_t}.
\end{aligned}
$$

Appendix A.2. Decomposition of Risk Into Traded and Non-Traded Components

$$\begin{aligned}
\mu &= \frac{E[CFADS_t]}{CFADS_{t-1}} - 1 = \frac{E[CFADS_t^T] + E[CFADS_t^N]}{CFADS_{t-1}} - 1 \\
&= \frac{CFADS_{t-1}^T}{CFADS_{t-1}} \frac{E[CFADS_t^T]}{CFADS_{t-1}^T} + \frac{CFADS_{t-1}^N}{CFADS_{t-1}} \frac{E[CFADS_t^N]}{CFADS_{t-1}^N} - 1 \\
&= w_{t-1}^T(1+\mu^T) + w_{t-1}^N(1+\mu^N) - 1 = w_{t-1}^T\mu^T + w_{t-1}^N\mu^N \\
&= w_{t-1}^T(r + \lambda^T\sigma^T) + w_{t-1}^N(r + \lambda^N\sigma^N) \\
\Rightarrow \mu &= w_{t-1}^T\frac{\sigma^T}{\sigma}\lambda^T + w_{t-1}^N\frac{\sigma^N}{\sigma}\lambda^N,
\end{aligned}$$

Appendix B. Risk Measures

Here we outline the calculation of risk and return measures used in this paper.

Appendix B.1. Credit Risk

Expected loss is computed as

$$E_t^*[\text{Loss}] = \sum_{i=t}^{T} e^{-r(i-t)} \left(\text{DS}_i^{\text{BC}} - E^*[\text{DS}_i]\right), \tag{B.1}$$

where $E^*[\text{DS}_i]$ is the mean debt payment for the i^{th} period computed under the risk neutral probability measure. We compute expected losses under the risk neutral measure so that the present value of expected losses is influenced not only by mean losses, but also by the distribution of losses around the mean level.

The percentage expected loss $E_t^*[l]$ and recovery rate $E_t^*[RR]$ can be written as

$$E_t^*[l] = \frac{E_t^*[\text{Loss}]}{\sum_{i=1}^{T} e^{-r(i-t)} E^*[\text{DS}_i]}, \tag{B.2}$$

$$E_t^*[RR] = 1 - E_t^*[l]. \tag{B.3}$$

Appendix B.2. Interest Rate Risk

Duration is calculated as (Tuckman, 2002)

$$D_t = \frac{-1}{V^D(t)} \frac{\partial V^D(t)}{\partial y_t}$$
$$\Rightarrow D_t = \frac{1}{V^D(t)} \sum_{i=t+1}^{T_D} (i-t) e^{-y_t(i-t)} \mathrm{DS}_i^{\mathrm{BC}} \tag{B.4}$$

where $V^D(t)$ is the value of the debt at time t, and y_t is the yield at time t.

Appendix C. Return Measures

The yield y_t and z-spread s_t are defined as

$$V_t^D = \sum_{i=t}^{T_D} e^{-y_t(i-t))} \mathrm{DS}_i^{\mathrm{BC}}, \tag{C.1}$$

$$V_t^D = \sum_{i=t}^{T_D} e^{-(r_{t,i}+s_t)(i-t))} \mathrm{DS}_i^{\mathrm{BC}}. \tag{C.2}$$

Appendix D. Data Collection Requirements

Our methodology only requires a parsimonious dataset as input. The key model inputs needed to calibrate the distribution of $DSCR_t$ are given in table D.3.

This data is routinely collected by lenders in project finance either at financial close since it is part of the final financial model or during the life of the loan. In this case, we only need to collect data on realised DSCR values at each point in time, as well as to answer a number of simple questions about which state the SPE is in at that time (e.g. default, lock-up, bankrupt etc.)

Appendix E. Acknowledgements

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Table D.3: Data collection requirements.

stage	data points
ex ante (financial close)	- Base case debt service and calendar
	- Base case $CFADS_t$ (optional), $DSCR_t$, ADSCR
	- Covenants (reserve accounts, cash sweep, default triggers, etc)
	- Initial senior and subordinated debt, initial equity
	- Foreign exchange mismatch (y/n), interest rate swap (y/n)
	- Project dates, life, construction start and completion dates
	- Country, sector, revenue risk profile, input risk
One-off events	- First drawdown (date)
	- First debt service payment (date)
	- Construction start (date)
	- Construction completion (date)
ex post (status at time t)	- $DSCR_t$
	- Lockup at time t (y/n)
	- Technical default at time t (y/n)
	- Hard default at time t (y/n)
	- Bankrupt at time t (y/n)
	- Refinancing at time t (y/n)
	- Emergence from default at time t (y/n)

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