Measuring the credit risk of unlisted infrastructure debt

Theoretical framework and cash flow reporting requirements

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Abstract

In this paper, we develop a framework to measure the credit risk of unlisted infrastructure debt, including the first formulation of "distance to default" in infrastructure project finance.[1]

We propose to use the debt service cover ratio (DSCR or the ratio of the firm's free cash flow to its debt service in a given period), which is routinely collected by project finance lenders, to measure and benchmark credit risk in infrastructure project finance.

We argue that knowledge of the first two moments of distribution of the DSCR in project finance are sufficient to measure and predict the credit risk of individual loans. We show that the distribution of the DSCR captures asset value and volatility and allows measuring *distance to default* in project finance. The distribution of the DSCR also provides an unambiguous default point and can thus be used to build a mapping of expected default frequencies (EDFs) in project finance.

Once characterised, the distribution of the DSCR allows the computation of an expected value, a conditional probability of default at time *t* and a conditional probability of emergence from default.

We show that these variables are sufficient to compute loss given default (LGD) and the expression of a loss density function of project finance loans at each point in the project lifecycle. Thus, the knowledge of the distribution of the DSCR in project finance allows the calculation of a value-at-risk (VaR) measure of infrastructure project debt, which can be used, for example, to calibrate a risk module, such as those used in risk-based prudential frameworks.

We highlight the relevance of our conclusions with an illustrative simulation.

We also conclude that a large sample of observed or simulated project debt cash flows and their respective DSCR in each period, could be used to derive either a functional form for the distribution of the DSCR or an empirical mapping of distance to default and probabilities of default in project finance *at each point in the project lifecycle*.

Thus, we propose a template to support data collection initiatives and improve the future benchmarking and transparency of infrastructure investments.

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1 - Our approach focuses on single debt instruments. A portfolio/benchmarking approach will be developed in a future paper, including the discussion of appropriate portfolio risk measures, the role of correlations and the choice of strategies available.

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1 Introduction

The fallout of the 2009 financial crisis has triggered a slow but certain paradigm shift for the asset allocation decisions of institutional investors. The objective of diversifying away from market volatility along with the increasing role played by liability-driven investment is fuelling increasing interest in unlisted assets with long-dated maturities, predictable cash flows and attractive yields.

Infrastructure investment is amongst the areas that intuitively offer some of these appealing characteristics. Infrastructure debt is the main candidate for new allocations since it is useful both from a diversification and asset-liability management perspective for an insurer or pension fund. Moreover, as we discuss in a recent paper (Blanc-Brude, 2013), most infrastructure financing consists of debt financing. Hence, infrastructure debt is the most relevant area of investment from an institutional perspective.

However, the investment profile of these assets is not well documented and often ill-understood. Today, despite a few empirical studies, there does not exist any scientific benchmark of unlisted infrastructure debt credit risk.

In this paper, we develop a framework to measure the credit risk of unlisted infrastructure debt, including the first formulation of "distance to default" in infrastructure project finance. Section 2 describes our intuition: we propose to use the debt service cover ratio (DSCR or the ratio of the firm's free cash flow to its debt service in a given period), which is routinely collected by project finance lenders, to measure and benchmark credit risk in infrastructure project finance.

Our intention is to develop risk measures for infrastructure debt that are both rooted in modern financial theory and implementable empirically because we know that the necessary data can be collected.

Hence, we argue that knowledge of the first two moments of distribution of the DSCR in project finance are sufficient to measure and predict the credit risk of individual loans. In section 3, we show that the distribution of the DSCR captures asset value and volatility and allows measuring *distance to default* in project finance. The distribution of the DSCR also provides an unambiguous default point and can thus be used to build a mapping of expected default frequencies (EDFs) in project finance.

Once characterised, the distribution of the DSCR allows the computation of the expected value $E(DSCR_t)$, the probability of default $p_t = \Pr(DSCR_t < 1.x | min_{j<t} DSCR_j \geq 1.x)$ and the probability of emergence from default $q_t = \Pr(DSCR_t \geq 1.x | DSCR_{t-1} < 1.x)$.

In section 4, we show that these variables are sufficient to compute loss given default (LGD) and the expression of a loss density function of project finance loans at each point in the project lifecycle. Thus, the knowledge of the distribution of the DSCR in project finance allows the calculation of a value-at-risk (VaR) measure of infrastructure project debt, which can be used, for example, to calibrate a risk module, such as those used in risk-based prudential frameworks.

We highlight the relevance of our conclusions with an illustrative simulation in section 5.

Finally, we conclude that a large sample of observed or simulated project debt cash flows and their respective DSCR in each period, could be used to derive either a functional form for the distribution of the DSCR or an empirical mapping of distance to default and probabilities of default in project finance *at each point in the project lifecycle*.

For this purpose, in section 7, we propose a data collection template to built facilitate the participation of investors and lenders to a data collection effort and improve the future benchmarking and transparency of infrastructure debt investments.

2 Intuition

In this section, we argue that all necessary infrastructure debt credit risk metrics can be derived from the first two moments of the statistical distribution of a project finance *debt service cover ratio*.

2.1 Defining infrastructure debt

2.1.1 Infrastructure debt is (mostly) unlisted project finance loans

What constitutes "infrastructure debt" remains to be universally defined. In this paper, we do not concern ourselves with the oft-mentioned *project bonds*, partly because they remain rare and therefore of little relevance from a strategic asset allocation perspective, and partly because they are capital market instruments, which is at odds with one of the primary motives of institutional investors considering investments in infrastructure: diversifying away from market volatility into unlisted, possibly illiquid assets that also have an attractive yield.[2]

Instead, we focus on the measurement of credit risk for **unlisted senior loans extended to large investment projects on a limited recourse basis** i.e. *project finance*.

Our focus on project finance is warranted, in the first instance, because most infrastructure investment and the immense majority of new or `greenfield' investments are delivered via project financing[3]; and second, because contrary to the ill-defined notion of `infrastructure investment', project finance benefits from a clear and universally recognised definition since the Basel-2 Capital Accord.

> "Project finance is a method of funding in which investors look primarily to the revenues generated by a single project, both as the source of repayment and as security for the exposure. In such transactions, investors are usually paid solely or almost exclusively out of the money generated by the contracts for the facility's output, such as the electricity sold by a power plant. The borrower is usually a Special Purpose Entity that is not permitted to perform any function other than developing, owning, and operating the installation. The consequence is that repayment depends primarily on the project's cash flow and on the collateral value of the project's assets." (BIS, 2005)

Hence, by focusing on project finance, we capture the bulk of private infrastructure financing and gain **a clear definition of infrastructure debt at the underlying level**. This is instrumental since our purpose is to discuss infrastructure investment on a scale that is congruent with institutional investing i.e. implying substantial asset holdings. To achieve a degree of generality in our conclusions, especially from an asset allocation perspective, we must focus on the most representative investment format.

2.1.2 The role of debt in project finance

Project financing amounts to investing in a single-project firm or SPE with a pre-defined lifespan. Before the financing decision can been taken, the SPE has to demonstrate its financial viability with a high degree of probability. In the process, two inter-related types of financial claims are created, **splitting the** ***free***

2 - As we have identified in previous publications, (see for example Blanc-Brude and Ismail, 2013a)
3 - We estimate that more than USD3Tr of project financing was closed worldwide between 1995 and 2012 (Blanc-Brude and Ismail, 2013b).

cash flow[4] **of the project between a senior, fixed-rate claim on one hand, and subordinated, fixed-rate and variable rate claims on the other**[5] (see Blanc-Brude and Ismail, 2013b).

Taken as a whole, the claims that constitute an instance of project financing can be interpreted as a portfolio of inter-linked bonds, with different maturities and grace periods, some paying a fixed rate of interest and some paying a variable rate of interest.

The analogy with a portfolio of bonds or cash flows is justified by the fact that in the majority of cases, the project SPE does not own any tangible assets[6], or owns assets that are so *relationship-specific* [7] that they have little or no value outside of the contractual framework that justifies the investment.

Limited recourse financing also means that the owners of the SPE provide very little, if any collateral to secure the its debt. In project finance, contracts must suffice to create enforceable and valuable claims and to define expected cash flows with reasonable accuracy (see Blanc-Brude, 2013, for a discussion).

Finally, an important feature of project finance is the role of *initial* financial leverage (agreed at financial close). In a recent review, we report that senior leverage[8] in infrastructure project finance consistently averages 75% between 1994 and 2012, irrespective of the business cycle, and can be as high as 90% (Blanc-Brude and Ismail, 2013b).

We and others have argued that the high leverage typically observed in project finance should be interpreted as a sign of *low* asset risk (Esty, 2003; Blanc-Brude, 2013) i.e. lenders agree to provide most of the funds necessary to carry out the planned investment without further recourse or security because the probability of timely repayment is considered to be very high. In other words, the `split' of the project's free cash flow between senior (low risk) and junior (riskier) instruments suggested above, results in a larger senior tranche if cash flows are more predictable.

2.2 Nature of the underlying asset

Without substantial sponsor guarantees (limited recourse) or any tangible assets collateral (relationship-specific capital investment), the only source of present or future value at the level of the project SPE is its free cash flow, or the cash flow available once the various tasks that the SPE has contractually committed to accomplish in each period (e.g. build, maintain and operate a large structure) have been executed. There is no terminal value (TV).

Thus, in limited recourse project financing, as opposed to traditional corporate finance, the free cash flow of the firm is the main determinant of asset value. At any time t during the SPE's finite life, the firm's **asset value** is simply the sum of expected Cash Flow Available for Debt Service or CFADS, discounted at the appropriate rate. This expected value is the only quantity against which the SPE may initially borrow (or later re-finance) any debt.

4 - or net operating cash flow

5 - The senior debt may also be characterised as "fixed-spread", while the actual interest rate is benchmark rate plus the spread. The junior claim receiving a variable spread.

6 - In the most frequent case of public infrastructure projects financed through a so-called public-private partnership contract, the ownership of the tangible infrastructure assets remains *de facto* and, most often, *de jure* in the public domain

7 - Relationship-specific assets have very few if any alternative uses e.g. a coal terminal at the end of a single railway line leading to a coal mine in an otherwise sparsely inhabited part of Eastern Australia

8 - The ratio of senior debt to total investment

The CFADS thus plays a central role in our approach to risk measurement and the valuation of infrastructure debt. It is risky (stochastic) and can intuitively be understood as the *underlying process* driving value and risk in project finance debt securities, not dissimilar the underlying stochastic processes referred to in the design of asset pricing formulas.

Moreover, in project finance, the CFADS can be monitored by lenders on an ongoing basis, and provides a direct measure of the firm's asset value and volatility.

2.3 Debt service cover ratio and the default point

2.3.1 *Ex ante* DSCR

The relationship between the CFADS and the expected senior debt service i.e. the ability of a given SPE to service its senior debt obligation, is captured by a **debt service cover ratio** (DSCR), which is routinely calculated by project finance lenders for each SPE.

The DSCR at time *t* is written:

$$DSCR_t = \frac{\text{Cash Flow Available for Debt Service (CFADS)}_t}{\text{Debt Service (Principal+Interest)}_t} \tag{1}$$

in each period *t*=1,2,..T for a project financing of maturity T.

Gatti (2012) reports that average *ex ante*[9] DSCRs typically range between 1.35 and 1.40. As SPEs meet their senior debt obligations, their DSCR can be expected to evolve over time even though lenders may structure the debt amortisation profile in such a way that the *ex ante* DSCR is constant across the lifecycle, as is often the case in social infrastructure projects with a guaranteed income stream[10].

We draw two conclusions from the definition of the DSCR.

First, as a function of the CFADS i.e. the underlying process explaining firm value, the distribution of the DSCR in project finance captures both expected asset values and volatility. We discuss this point in more details in section 2.4.

Second, the DSCR provides an *unambiguous* definition of default, which we discuss next.

2.3.2 *Ex post* DSCR and the default point

A "hard" default of the SPE i.e. an actual default of payment, can be defined in terms of the *ex post* CFADS at time *t*, as:

$$\text{Default}_t \Longleftrightarrow \text{CFADS}_t < \text{Debt Service (Principal+Interest)}_t \tag{2}$$

which can be expressed in terms of *ex post DSCR* as:

$$\text{Default}_t \Longleftrightarrow DSCR_t \equiv \frac{\text{CFADS}_t}{\text{Debt Service (Principal+Interest)}_t} < 1 \tag{3}$$

9 - For our purpose, it is useful to distinguish between the *ex ante* and *ex post* values taken by the cash flows and their associated ratios. *Ex ante* cash flows correspond to the values agreed on in the project's financial model at financial close. *Ex post* cash flows and ratios denote the realised values during the project's life, simulated or observed.

10 - e.g. the Private Finance Initiative in the UK.

By definition, if the *ex post* DSCR equals unity, the SPE is just able to service its senior debt during the relevant period, and if it falls below unity, the borrower can *unambiguously* be considered in default.[11]

However, default events may also be defined more loosely. For example, in the Basel-II framework, project finance default is defined as '*...past-due more than 90 days on any material credit obligation to the banking group*' (BIS, 2005).

Indeed, lenders typically require the *ex ante* DSCR to be significantly higher than unity in order to create a credit risk buffer. If the *ex post* DSCR is too low, debt holders can impose the so-called "lock up" of equity distributions, until debt service coverage returns to a pre-agreed level.[12] A low *ex post DSCR* may thus constitute a breach of the loan's covenants and also be considered an event of default.

Hence, the default point in project finance at time t can be defined as:

$$DSCR_t = 1.x \text{ with } x \geq 0$$

And since the DSCR provides an unambiguous estimate of the default point of infrastructure project finance debt, its probability of default at time t can be written:

$$p_t = \Pr\left(DSCR_t < 1.x | min_{j<t} DSCR_j \geq 1.x\right)$$

i.e. it is the probability that the DSCR reaches the default point conditional on there having been no default until that time.

Next, in light of the conclusions above, we discuss the relevance of structural credit risk models in the analysis of project finance credit risk.

2.4 A structural approach to project finance credit risk

We argued above that in limited recourse project financing, the CFADS can be interpreted as an underlying process driving asset value and volatility, both of which can be measured by monitoring the DSCR, which also provides an unambiguous definition of the default point.

Hence, structural models of credit risk can be expected to provide useful insights into project finance credit risk. Indeed, approaching debt as a derivative written on an underlying asset value requires estimating asset value, volatility and a default point.

2.4.1 The relevance of structural models

Structural models postulate the existence of a default triggering mechanism i.e. a discrete event at the threshold between two states (default vs. no default). In other words, default events are not random amongst firms but must result from a contractual or financial breach.

In academic finance, such models are expansion of the work of Black-Scholes (1973) on pricing firm equity as a call option on the company value, as well as Merton (1974), Black and Cox (1976) and Ingersoll (1977) who established what is now referred to as Merton model.

11 - Moody's definition of project finance default as '*a missed or delayed disbursement of interest and/or principal...*' Moody's (2013) is congruent with this view.

12 - The DSCR should also on average be higher than unity so that equity/junior distributions can be made once senior debt obligations have been met

In the Merton model, the value of the company follows a stochastic process (V_t). The company is financed from debt and equity, its debt is a single obligation and resembles a zero coupon bond with face value B and maturity T. At time t, the value of the firm is the sum its equity S_t and debt B_t ($V_t = B_t + S_t$, for $0 < t < T$).

Under this model, the firm does not pay any dividends nor issues any new debt. If at maturity the value of the firm is less than its liabilities ($V_T < B$), the firm is considered to be in default. The equity holders then choose not to provide any new equity capital as an expression of their 'limited liability option' and hand over the firm to the debt holder, which liquidates the remaining assets and receive the proceeds $B_T = V_T$.

If there is no default, the debt holder receives the payoff B, and equity holders receive the remaining of the firms value $V_T - B$.

It is now a classic result that this model implies for the value of the firm's equity at time T to be equivalent to the payoff of a European call option on V_T, while the debt value equals the nominal value of liabilities (as risk free zero coupon bond) less the payoff of a European put option on V_T. Under a number of assumptions,[13] there is a closed form solution for the value of the firm's debt, which can be priced as the value of standard plain vanilla options (McNeil et al., 2005).

As is well-documented in the literature, this model has been criticised for making a number of assumptions, including the lognormal distributions of returns and a simplistic capital structure (the firm borrows once and subsequently de-leverages). The definition of default used in the Merton model has also been criticised: the default point is not only assumed to be known unambiguously (when asset value falls below liabilities) but the firm *must* default exactly when this point is reached, neither of which is self-evident empirically.

However, it should be clear from the discussion above that the Merton model is rather well-suited to project financing: SPEs borrow once and subsequently de-leverage, default is unambiguously known and actively monitored, and the underlying process driving asset value (CFADS) can be captured by the distribution of the DSCR.

2.4.2 Significance of the DSCR distribution

Simply put, lending to a project SPE can be portrayed as the equivalent of writing a derivative contract on the project's CFADS with the *ex ante* agreed debt service as the strike price.

As argued above, in limited-recourse project financing, the discounted cash flow available for debt service (CFADS) is equivalent to the firm's asset value, and $CFADS_t$ can be described as a stochastic process explaining the change in asset value in each period.

$DSCR_t$, the ratio of the $CFADS_t$ to the expected debt service at time t, captures the information that is both *necessary and sufficient* to value infrastructure project debt: its expected value is a function of asset value, its standard deviation is a measure of the volatility of asset value, and its distribution can be used to derive the conditional probability of reaching the default point defined above at time t i.e. $\Pr\left(DSCR_t < 1.x \middle| min_{j<t} DSCR_j \geq 1.x\right)$.

13 - Including that the firm's value process follows a lognormal distribution

In the rest of this paper, we will show that if a robust functional form can be established for $DSCR_t$ i.e. *at each point in the project lifecycle*, it is sufficient to derive the necessary credit risk metrics of infrastructure project finance debt. Alternatively, without assuming its functional form, the empirical distribution of $DSCR_t$ can be used to build a mapping of expected default frequencies (EDF) in project finance. We return to both approaches point in section 3.

In effect, $DSCR_t$ is akin to the concept of "distance to default" (DD) already defined in the credit literature (KMV, 1993). We develop this point next, in section 3.1.

Finally, we will show in section 4 that the knowledge of the distribution of $DSCR_t$ also allows the measurement of loss given default and *in fine*, the expression of a loss density function for project finance loans, which can be used, for example, to calibrate a dedicated risk module with a value-at-risk (VaR) measure.

3 Predicting probabilities of default

In the structuring process of infrastructure project debt, the amortisation profile is designed to deliver an expected value of the DSCR, which is necessarily higher than unity[14]. Credit risk in project finance arises because the *ex post* DSCR may be different from its *ex ante* value.

The notion of distance to default (DD), derived from structural credit models, is the number of standard deviations required for a firm to reach its default point within a specified time horizon t. Computing DD_t requires estimating asset value, volatility and the default point at the relevant horizon, and it is often put forward in the credit literature as a sufficient statistic to provide a rank ordering for publicly traded firms default risk: the higher DD_t, the less likely it is for a firm to default at time t (KMV, 1993).

As far as we know, the notion of distance to default has never been applied to unlisted loans like infrastructure project finance debt. However, DD_t is intuitively linked to $DSCR_t$ and we have argued in the previous section that the distribution of $DSCR_t$ is sufficient to derive a volatility measure of the underlying asset and an unambiguous estimate of the default point.

In this section, we re-formulate the DD measure for an individual project as a function of its DSCR distribution and the base case debt service. We show that the knowledge of the first two moments of the $DSCR_t$ distribution (mean and variance) is sufficient to derive DD_t for individual loans.

3.1 Distance to default

In the Merton model, the firm's asset value follows a lognormal process with expected growth rate μ and its asset volatility is σ_V, for $t = 1$. Hence, the probability of default for an initial firm's asset value of V_0 is

$$\text{probability of default} = \Phi\left[-\frac{\ln(\frac{V_0}{B}) + (\mu - \frac{1}{2}\sigma_V)}{\sigma_V}\right] \tag{4}$$

Drawing from the Merton model, the KMV model defines the negative of the quantity inside the brackets as the Distance to Default or DD KMV (1993). The default probability is the area under the distribution below the DD point.

$$\text{probability of default} = \Phi\left[-DD\right] \tag{5}$$

Using this definition, together with a number of assumptions, KMV proposes at a simpler expression for DD. For default point value of $\bar{B}$, DD is given by:

$$DD := \frac{\ln(V_0) - \ln(\bar{B})}{\sigma_V} \tag{6}$$

which can be approximated as (McNeil et al., 2005):

$$DD \approx \frac{V_0 - \bar{B}}{\sigma_V V_0} \tag{7}$$

14 - This is usually referred to as the "bank base case"

or, without loss of generality (Crosbie and Bohn, 2003):

$$\text{Distance to Default} = \frac{[\text{Market value of assets}] - [\text{Default point}]}{[\text{Market value of assets}].[\text{Asset volatility}]} \tag{8}$$

where the asset volatility is the **standard deviation of the annual percentage change in the asset value**.

The KMV model premises that DD is a sufficient statistic to arrive at a rank ordering of default risk, where the numerator in (6) expresses the firm's financial leverage or *financial risk*, while the denominator reflects its *business risk*. In other words, KMV assumes that differences between companies are reflected in their asset values, asset volatilities and capital structure, which are all incorporated in the DD measure (Kealhofer, 2003) and that firms with equal DD must have equal default probabilities[15].

When applying the KMV model, the company's unobservable asset value and asset volatility are inferred using observable equity market returns and the default point is assumed to be the value of its short liabilities and half of its long term liabilities value for financial firms, and a percentage of total adjusted liabilities for non-financial firms.

In effect, the DSCR in project finance reflects the *financial risk* of the investment driven by (de)-leveraging, as well as the *business risk* implied by the volatility of the CFADS.

Following the definition of default in project finance given in (2), Distance to Default for infrastructure project finance loans at time *t* can be defined as:

$$DD_t = \frac{\text{CFADS}_t - \text{Debt Service}_t}{\sigma_{\text{CFADS}_t}\text{CFADS}_t} \tag{9}$$

Using the definition of $DSCR_t$ in (1), the above expression can be written as:

$$DD_t = \frac{1}{\sigma_{\text{CFADS}_t}}(1 - \frac{1}{DSCR_t}) \tag{10}$$

(10) can be re-written as a sole function of $DSCR_t$ by expressing the volatility of $CFADS_t$ as a function of that of $DSCR_t$.

We have $CFADS_t = DSCR_t \times \text{Debt Service}_t$, and we know that σ_{CFADS_t} is expressed as a percentage change in the asset value in (10), thus:

$$\sigma_{\text{CFADS}_t} = \sigma_{DSCR_t}\frac{\text{Debt Service}_t}{\text{Debt Service}_{t-1}}$$

Replacing in (10), we have:

$$DD_t = \frac{1}{\sigma_{DSCR_t}}\frac{\text{Debt Service}_{t-1}}{\text{Debt Service}_t}(1 - \frac{1}{DSCR_t}) \tag{11}$$

where σ_{DSCR_t} is the standard deviation of the annual percentage change in the *DSCR* value.

Hence, **the distribution of $DSCR_t$ together with the debt repayment profile (growth rate of the debt service) are sufficient inputs to estimate the Distance to Default of project finance loans**.

15 - The KMV model also introduces a number of modifications to the basic Merton's model, such as assuming that default can happen at any time, the empirical determination of the default point ($\bar{B}$), and allowing for cash payouts.

3.2 Expected default frequencies in project finance

DD only provides a rank ordering of default risk and moving from DD to quantifying probabilities of default requires either parametric assumptions regarding the underlying process driving asset value (c.f. Merton model), or the empirical mapping of the relationship between DD and probabilities of default as proposed by KMV. We discuss both in turn below, as well as the expected role of the project lifecycle on credit risk.

3.2.1 A functional form for $DSCR_t$

The most powerful implementation of a structural model relies on assuming and calibrating a functional form for the underlying process. In our case, any functional form taken by the CFADS is reflected in the $DSCR_t$. We may thus assume a functional form for $DSCR_t$.

A key question is to know whether there is a single, unimodal distribution of $DSCR_t$ for all projects or if the different moments of the distribution are themselves determined by a series of factors.

This is an empirical question, which we return to in section 6.2. Existing empirical studies of default probability in project finance (Moody's, 2013; Standard & Poor's, 2004) content themselves with measuring the number of observed default events during a given period within a population of loans. As such, they assume a binomial distribution of default rates. In other words, they assume that each loan is equally likely to default within the considered time period.[16] However, it is unlikely that a single credit risk profile exists for all infrastructure project loans and the reported probabilities of default may require qualification.

We have argued before that industrial sector classifications (e.g. roads, electricity &c.) are unlikely to explain the risk profile of infrastructure investments (Blanc-Brude, 2013) i.e. any attempt to weight the distribution of default frequencies by industrial sector is expected to fail a test of goodness of fit. Instead, we expect specific risk factors, in particular, revenue risk factors to have the most significant impact on credit risk.

More recent empirical studies address the dynamic dimension of project finance credit risk and compute marginal default rates (in each year from origination). They report that average default probabilities in project finance continuously decline throughout the project lifecycle (Moody's, 2013, 2012). However, rather than the passage of time, this observed credit migration may be better explained by changes in the level and variance of $DSCR_t$.

We currently do not have the answers to such questions about the empirical distribution of $DSCR_t$ in project finance. In section 6.2, we discuss the need to systematically collect data on the risk factors that may explain the distribution of $DSCR_t$ and therefore be instrumental in the benchmarking of infrastructure debt.

Next, we describe the alternative approach to assuming and calibrating a functional form for $DSCR_t$.

3.2.2 Empirical mapping of EDFs for project finance loans

The empirical mapping of default rates from DD measures has been developed by KMV for publicly traded companies using data on historical default and bankruptcies. Once the mapping has been built with a

16 - This is usually referred to as the "naive" definition of probability: a set contains a finite number of outcomes and every experiment in the set is equally likely (Blitzstein, 2006)

satisfactory level of fit, DD measures can be used to derive forward-looking default probabilities within a given time period i.e. expected default frequencies (EDF).

Moody's uses an empirical default database, with more than 8,600 defaults as of the end of 2011, to build functional relationship or mapping between DD and EDFs relying on historical data. Kealhofer (2003) reports a strong empirical relationship between *DD* and observed default rates.

Following this approach, using either an observed or assumed distribution of $DSCR_t$, a complete mapping of DD_t and EDF_t may be established from $DSCR_t$ in project finance.

As an illustration, we discuss a simulated mapping of project finance EDFs in section 5.

3.2.3 A dynamic risk profile

When applied to listed firms, the KMV model is used to predict default frequencies within a single period (e.g. the following 12 months). Any multi-period estimate of DD entails increasing estimation errors of asset prices and volatility.

In the case of project finance however, credit risk is dynamic (i.e. follows a predictable migration). Contrary to publicly traded firms, project finance SPEs have a finite life and each period in their lifecycle can be characterised *ex ante*: as have argued on theoretical grounds in a previous paper (Blanc-Brude and Ismail, 2013b), the risk profile of infrastructure project finance debt is dynamic because of the ongoing de-leveraging of the single-project firm or SPE. Sorge (2004), following an insight from Merton (1974) suggests that two effects impact long-term credit risk in project finance: longer maturities are less likely to be repaid but continued de-leveraging has the opposite effect. For firms with a high level of initial leverage, the later effect can be strong enough to offset the impact of the long-term on credit risk.

Whether a functional form is assumed for the distribution of $DSCR_t$ or an empirical mapping is built form observed DSCR values, the dynamic nature of the underlying process should be addressed in the modelling of infrastructure project debt credit risk.

With a continuously de-leveraging firm, $DSCR_t$ may, *ceteris paribus*, be expected to be an increasing function of time. However, we also know from practice that project finance debt service can be structured (sculpted) to deliver a constant *ex ante* DSCR. The endogenous nature of credit risk in project financing explains its dynamic nature (see Blanc-Brude and Ismail, 2013b, for a discussion).

This opens a number of empirical questions to be addressed in due course to model project finance credit risk, including whether there is any evidence of heteroscedasticity in $DSCR_t$ (non-constant variance) . Hence, empirically, we must consider the determinants of $DSCR_t$ both in the cross-section and in time. We return to this point in section 6.2.

3.3 Emergence from default

Finally, we know from the empirical literature on project finance that events of default rarely lead to liquidation i.e. the lender exercising its option to capture the remaining asset value (recovery value) in the defaulted firm. Instead, there is a documented tendency for lenders and borrowers to 'work out' a restructuring and for SPEs to *emerge* from default.

The finite life and dynamic credit profile of project finance structures provide a greater visibility for lenders who need to decide whether to allow a restructuring or to liquidate the firm. Through a restructuring, lenders may in fact increase the value of the debt by imposing new conditions such as cash sweeps[17].

Given the significant role played by workouts and restructurings in the event of default in project finance, any assessment of credit risk should incorporate this dimension.

Again the knowledge of the distribution of the $DSCR_t$ is sufficient to derive the necessary statistics, since the probability of emergence from default at time t or q_t, is simply the probability of observing a DSCR higher than the default point in a given period, conditional to having observed a DSCR below the default point in the previous period, or:

$$q_t = \Pr(DSCR_t \geq 1.x | DSCR_{t-1} < 1.x)$$

Having determined the role of $DSCR_t$ in explaining and predicting probabilities of default and emergence from default in project finance, we now turn to the estimation of *loss given default* in section 4.

17 - All CFADS must be used to pay down debt for a given horizon.

4 Loss given default

In this section, we propose an expression of both the value and loss functions of infrastructure project finance debt in terms of the DSCR. The loss function allows the calculation of different quantile-based risk measures such as the 99.5% value-at-risk (VaR) or the expected shortfall measure (conditional VaR) of project finance debt.

Finally, we discuss the question of discounting and propose a short solution for the purpose of the empirical illustration in section5. We will discuss the derivation of the appropriate discount rates for project finance debt in a future paper.

4.1 Value and loss functions

4.1.1 Payoff given default

In the structural framework described earlier, the recovered amount (recovery) once the SPE defaults at a given time, is simply the CFADS at that time. Indeed, there is no recourse to the SPE shareholders and no TV.

Hence, if default occurs at time t, the payoff C at time t is:

$$C_t = \begin{cases} D_t & \text{with probability } (1 - p_t); \\ E(CFADS_t) & \text{with probability } p_t \end{cases} \tag{12}$$

for p_t is the probability of default at time t, conditional to no default prior to that time. As discussed in section 2.3.2, $p_t = \Pr(DSCR_t < 1.x | min_{j<t} DSCR_j \geq 1.x)$.

The cumulative discounted payoff is written:

$$\begin{aligned} V_0 &= \textstyle\sum_{t=1}^{T} \frac{1}{(1+r_t)^t}(D_t \times (1 - p_t) + E(CFADS_t) \times p_t) & (13)\\ &= \textstyle\sum_{t=1}^{T} \frac{1}{(1+r_t)^t}(D_t \times (1 - p_t) + E(DSCR_t) \times D_t \times p_t) & (14)\\ &= \textstyle\sum_{t=1}^{T} \frac{D_t}{(1+r_t)^t}(1 - p_t \times (1 - E(DSCR_t))) & (15) \end{aligned}$$

4.1.2 Payoff with emergence from default

We discussed the role of workouts in section 3.3. For simplicity, we limit our analysis to the possibility of emergence in the period immediately following default i.e. once default occurs at a given time t, there is a q_t probability of emerging from default by the beginning of $t + 1$. Expression (13) is now written:

$$\begin{aligned} V_0 &= \textstyle\sum_{t=1}^{T} \frac{1}{(1+r_t)^t}(D_t \times (1 - \frac{p_t}{1+p_{t-1}q_t/(1-p_{t-1})}) + E(CFADS_t) \times \frac{p_t}{1+p_{t-1}q_t/(1-p_{t-1})}) & (16)\\ &= \textstyle\sum_{t=1}^{T} \frac{D_t}{(1+r_t)^t}(1 - p_t(\frac{1}{1+p_{t-1}q_t/(1-p_{t-1})}) \times (1 - E(DSCR_t))) & (17) \end{aligned}$$

where, considering the emergence case, p_t is the probability of defaulting at time t conditional on surviving at previous time period, $q_t = \Pr(DSCR_t \geq 1.x | DSCR_{t-1} < 1.x)$.

Let $w_t = \frac{1}{1+p_{t-1}q_t/(1-p_{t-1})}$, the cumulative discounted payoff is:

$$V_0 = \sum_{t=1}^{T} \frac{D_t}{(1+r_t)^t}(1 - p_t \times w_t \times (1 - E(DSCR_t))) \tag{18}$$

where $0 < w_t \leq 1$.

4.1.3 Expected loss

So far, we have expressed the expected value of the cumulative payoff received for holding project debt from origination to maturity. The loss is defined as the difference between the face value of the project debt and its expected value. The loss function L is written:

$$L_0 = B_0 - V_0 \tag{19}$$

where, B_0 is the "base case" or *ex ante* debt service discounted at the relevant rate r_t

$$B_0 = \sum_{t=1}^{T} \frac{D_t}{(1+r_t)^t} \tag{20}$$

and V_0 is the expected payoff as per (18).

Hence,

$$L_0 = B_0 - \sum_{t=1}^{T} \frac{D_t}{(1+r_t)^t}(1 - p_t \times w_t \times (1 - E(DSCR_t))) \tag{21}$$

Loss expressed as a percentage of initial investment value is written:

$$\bar{L_0} = L_0/B_0 = 1 - \frac{1}{B_0}\sum_{t=1}^{T} \frac{D_t}{(1+r_t)^t}(1 - p_t \times w_t \times (1 - E(DSCR_t))) \tag{22}$$

4.1.4 Expected loss at time t

Finally, to account for the dynamic risk profile of project finance debt, we may want to compute a loss function across the lifecycle i.e. at each point during the maturity of the debt. This would also be useful in the hypothesis of the secondary market for infrastructure project debt.

Thus, conditional on no default prior to or at time t, the loss value at time $t > 0$ is written:

$$L_t = B_t - \sum_{i=t+1}^{T} \frac{D_i}{(1+r_i)^{(i-t)}}(1 - p_i \times w_i \times (1 - E(DSCR_i))) \tag{23}$$

Again, the distribution of $DSCR_t$ is instrumental in this setting since it includes information about both $E(DSCR_t)$, p_t and q_t. The other two inputs are the *ex ante* debt service D_t and the appropriate discount factors r_t, which we have not addressed so far and return to in section 4.2.

4.2 Discount rates

4.2.1 Choice of probability measure

The choice of discount rates is determined by the choice of probability measure for p_t and q_t.

Future debt cash flows can be discounted at the risk-free rate, for the relevant horizon, if their value is weighted by the corresponding risk-neutral probabilities of default i.e. those that would eliminate all possibilities of arbitrage between buyers and sellers of these cash flows (the risk-neutral measure). In structural models like the Merton model, the underlying value process is assumed to follow a stochastic process which is the result of efficient markets (no arbitrage condition).

However, when observing default rates in project finance debt empirically, we only know probabilities of default in the so-called *physical measure*. Because we do not know how far the physical measure lies from the risk-neutral measure (how efficiently the option written on the CFADS is priced) we cannot rely on risk-free discount rates and must examine the determinants of adequate discount factors.

4.2.2 Discount factor at t_0

We do not know the interest rates r_t term structure in (21). At this stage, we propose to use the **yield** y on the debt investment implied from the debt base case, as the appropriate discount factor approximating the interest rate term structure.

The yield is defined as the interest rate satisfying:

$$B_0 = \sum_{t=1}^{T} \frac{D_t}{(1+y)^t} \tag{24}$$

Given values for the initial investment B_0 and the debt service D_t, y can be derived.

The expected loss value at t_0 is written:

$$L_0 = B_0 - \sum_{t=1}^{T} \frac{D_t}{(1+y)^t}(1 - p_t \times w_t \times (1 - E(DSCR_t))) \tag{25}$$

4.2.3 Discount factors at time t

To calculate loss at time t, the associated series of yield to maturities has to be calculated first.

The series $\{B_t, y_t\}$ (for $t = 1, 2 \ldots, T - 1$) conditional on no prior default, can be calculated recursively employing the base case together with realized debt repayment at previous time period, starting with the known value of the investment at time $t = 0$, as well as the accompanying yield to maturity y_0 satisfying equation (24).

Given the investment value of B_t, the subsequent investment values can be calculated conditional on no prior default as:

$$B_{t+1} = (1 + y_t)B_t - \hat{D_{t+1}} \tag{26}$$

for $\hat{D_{t+1}}$ is the realized or actual debt repayment at time $t + 1$ [18].

This follows directly from the consistency condition according to which value at time t can be calculated as the discounted value one period ahead, combined with the discounted value of debt service payment at that time

$$B_t = \frac{D_{t+1} + B_{t+1}}{1 + y_t}$$

18 - If realized debt repayment $\hat{D_{t+1}}$ equals to the base case debt repayment D_{t+1}, it follows that the yield at time $t + 1$ equals to that of time t.

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The corresponding yield to maturity (y_t) can be calculated iteratively based on the investment value together with the base case cash flow payments. Hence,

$$B_t = \sum_{i=t+1}^{T} \frac{D_i}{(1+y_t)^{(i-t)}} \tag{27}$$

The loss function at time t is written:

$$L_t = B_t - \sum_{i=t+1}^{T} \frac{D_i}{(1+y_t)^{(i-t)}} (1 - p_i \times w_i \times (1 - E(DSCR_t))) \tag{28}$$

5 Illustrative simulation

Having established a methodology to derive distance to default, conditional probabilities of default and emergence from default, and a loss function for infrastructure project debt, we now propose a simple illustration of our results.

5.1 Approach and objectives

Our approach consists of assuming values for the mean and variance of $DSCR_t$, initial SPE leverage and a debt ammortisation profile. These variables are well-documented in project financing (see for example Gatti, 2012; Blanc-Brude et al., 2010).

While the simulation generates values at each point in the life of a population of senior project loans, it relies on assumptions, in particular about the volatility of $DSCR_t$, that are made at one point in time, here t_0. As such, this simulation represents a *prior* about the different states of the world that might affect loan repayments.[19]

The numerical simulation is set up thus: first, assuming a total investment normalised at 100, a base case debt service (principal and interest) D_t is derived from the proposed average leverage value and the choice of ammortisation profile.

Next, using the relationship between $DSCR_t$ and $CFADS_t$ described in (1) the mean and variance of $CFADS_t$ are derived. With these results, the simulation is performed assuming an assumed functional form for the distribution of $CFADS_t$.

Based on the distribution of the CFADS at time t, 100,000 Monte Carlo runs are performed to compare the values of D_t and $CFADS_t$ i.e. whether the simulated cash flow meets the debt service obligation of the SPE.

For each run, if the project defaults at time t as defined in (2), we consider the conditional probability of emergence in $t+1$ as defined in (3.3), which is a function of the distribution of $DSCR_{t+1}$. If the project does not emerge from default in at $t+1$, it is considered bankrupt and excluded form the next run. By definition the recovery value is $\text{E}(CFADS_t)$.

We calculate DD_t as defined in (10) and observe p_t, the probability of default at time t conditional on no prior default, and q_t the probability of emergence from default at time t conditional on default at time $t-1$.

Next, as described in section 4, for each run at time t, we calculate the base case value B_t using the yield from t to T as the discount rate, and the expected loss L_t as defined in (23), using the values obtained earlier for p_t, q_t and $\text{E}(DSCR_t)$. Finally, we calculate the 0.5% quantile of the distribution of L_t, that is, the 99.5% value-at-risk VaR_t.

19 - Once information about the realised states of the world becomes available (at t_2, t_3, &tc.) this prior can be revised or *updated* conditional on this new information. Here, we compute the probability of default at time t conditional on *simulating* no default until $t-1$, whereas *ex post*, it is possible to compute the probability of default at time t conditional on *observing* no default until time $t-1$.

Having made a (strong) assumption of about the functional form of the DSCR distribution, the outputs of the simulation are thus:

- DD_t: distance to default at time t, is computed according to equation (11);
- $p*_t$: the probability of default at time t, is computed as the ratio of simulated defaults ($DSCR_t < 1$) to that of surviving loans at time t;
- $q*_t$: the probability of emergence at time t is computed as the ratio of simulated defaults at time $t - 1$ to that of emerging defaults ($DSCR_t > 1$) at time t within the defaulted population at time $t - 1$;
- $E(L_t)$: the expected average loss at time t is computed according to equation (23);
- VaR_t: is the 0.5% quantile or 99.5% value-at-risk at time t of L_t.
- LGD_t: the average loss given default, that is $E(L_t | DSCR_t < 1)$

5.2 Assumptions

Table 1 summarises the main assumptions made in our simulation. As discussed above, the key assumption is to assume a functional form for the distribution of $CFADS_t$, in this case the lognormal distribution. It is a strong assumption that would not be required if sufficient empirical observations of $DSCR_t$ could be obtained, in which case a mapping of observed defaults and distance to default implied by the distribution of $DSCR_t$ could be derived without making further assumptions about the distribution of the underlying.

The second important assumption made is the mean value of $DSCR_t$, its rate of change and its variance. We examine two generic cases described in table 2.

1. **Increasing and increasingly volatile** $DSCR_t$: this is the most generic case of project financing. As the SPE de-leverages, its DSCR is expected to increase, however, revenue and costs also become more uncertain, resulting in a higher volatility of the DSCR as t increases. This profile corresponds to numerous projects which have an increasingly uncertain future, especially on the revenue side, but are also expected to de-risk with time, starting for their high initial leverage. Toll roads and power plant projects are typically structured this way. We label this case "generic economic infrastructure project", as illustrated on figure 1.

2. **Constant and stable** $DSCR_t$: the second case under consideration is more representative of the so-called "social infrastructure model" i.e. the SPE financing is structured so that the expected value of $DSCR_t$ is constant. This is frequent practice for school projects in the UK for example. A constant DCSR is a choice made by lenders in the structuration of the financing, which we interpret as signalling a constant *expected* risk profile (otherwise lenders can always structure a project so that the DCSR increases with time). If the risk profile is assumed to be constant, then distance to default must be constant, hence the volatility of $DSCR_t$ must be constant as well, as shown on figure 2.

5.3 Results

Two off-setting mechanisms drive the size of expected and extreme losses in senior infrastructure project debt:

- As the debt matures, asset value (the discounted sum of future cash flows) decreases and the *relative size* of a one-period loss (assuming emergence from default) increases.

Table 1: General assumptions

Variable	Assumption
CFADS distribution	Lognormal
SPE t_0 leverage	75%
Ammortisation profile	constant at an interest rate of 6%
Maturity	20 years
Average DSCR	1.4
Default	Can only happen once between t_1 and T
Emergence from default	Can only happen at t conditional on default at $t-1$

Figure 1: $DSCR_t$ expected value $\pm$ one standard deviation, generic economic infrastructure project debt

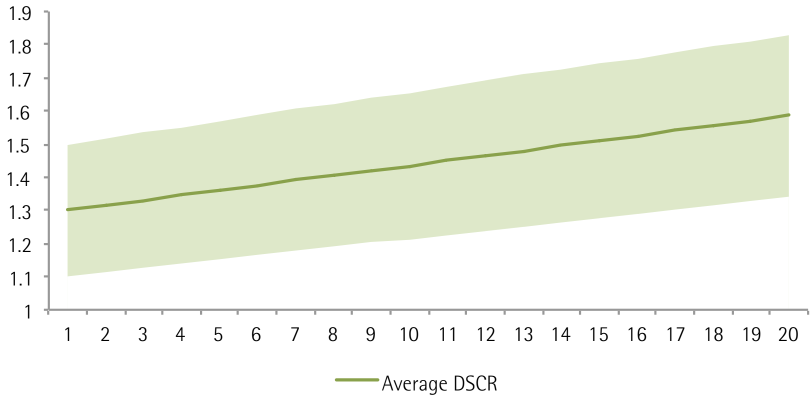


Figure 2: $DSCR_t$ expected value $\pm$ one standard deviation, generic social infrastructure project debt

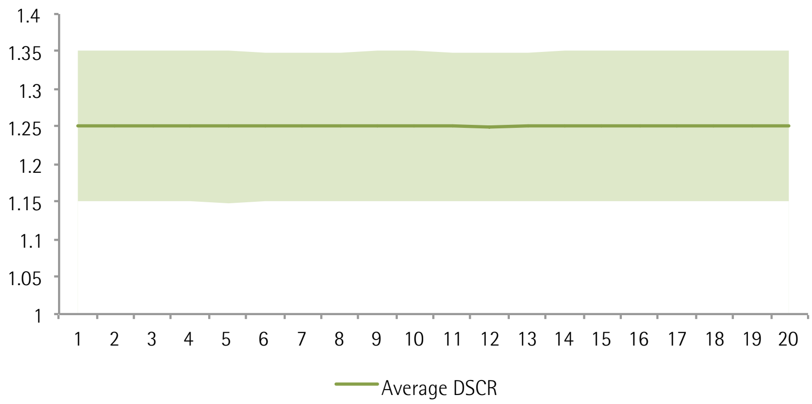


Table 2: $DSCR_t$ assumptions

Variable	Generic economic infrastructure project	Generic social infrastructure project
$DSCR_0$	1.3	1.25
$DSCR_T$	1.6	1.25
$\Delta DSCR_t$	linear from t_0 to T	no change
$\sigma DSCR_t$	0.2	0.1
$\sigma^2 DSCR_t$	0.04	0.01
$\Delta\sigma^2 DSCR_t$	+0.1%	no change

- At the same time, the number of potential future defaults decreases with the number of remaining periods until maturity. Since the expected loss is a function of all future potential defaults its *absolute size* decreases with time.

The combination of these two mechanisms can lead to a non-linearity in the risk profile if the balance of the two effects eventually reverses. We discuss this in more details below.

5.3.1 Generic economic infrastructure project

The credit risk profile of our generic economic infrastructure project financing are shown on figures 3, 4 and 5. The dynamic risk profile created by project financing results in increasing values of DD_t, despite the higher volatility of $DSCR_t$, falling probabilities of default with time and increasing probabilities of emergence from default.

As should be expected given our assumptions, there is a strong statistical relationship between DD_t and p_t as illustrated on 6. If the validity of the assumptions made about the functional form of the distribution of $DSCR_t$ can be confirmed empirically, then such a mapping can be used to predict default in project finance at different points in the project lifecycle.

Figure 7 shows an expected loss given default with a complex profile. During a first period, decreasing probabilities default during the loan's life result in lower *absolute loss* and thus lower expected loss given default. Towards the end of the loan's life, the effect of an increase in the *relative size* of the loss dominates and LGD_t increases sharply. During the last period, an investor who holds such a loan faces a higher loss given default than ever before.

Figure 8 shows that the VaR of project finance debt exhibits a "kink" after a few years. Initially the*relative loss* effect described above dominates but the rapid decrease in the probability of default (driven by the increasing $DSCR_t$) quickly shifts the balance in favour of the *absolute loss effect*, shrinking the size of extreme losses. In the last period, with a low default probability no more future cash flows to generate future potential losses, extreme losses are so rare that the 0.5% quantile is zero.[20].

Finally, we note that the results obtained from the simulation, both in terms of average probability of default and risk profile, are congruent with existing empirical research on default and recovery in project finance, including, for example, Moody's (2013).

5.3.2 Generic social infrastructure project

Figures 9, 10 and 11 show the results of the same simulation exercise assuming a constant and stable DSCR across the lifecycle of a generic social infrastructure project.

While this is likely to be a more restrictive setting than the previous case, this example highlights a different behaviour and provides the justification for empirical research and testing whether the DSCR can be considered to be drawn from one or several populations of project financings, as we discuss in section 6.2.

20 - As p_t decreases, q_t increases and B_t shrinks, the size and likelihood of extreme loss rapidly diminishIn a previous paper, we argue that such non-linear and dynamic risk profile is has strong implications for portfolio construction with infrastructure debt (see Blanc-Brude and Ismail, 2013b, for a detailed analysis).

Figure 3: Distance to default, generic 20-year economic infrastructure project debt

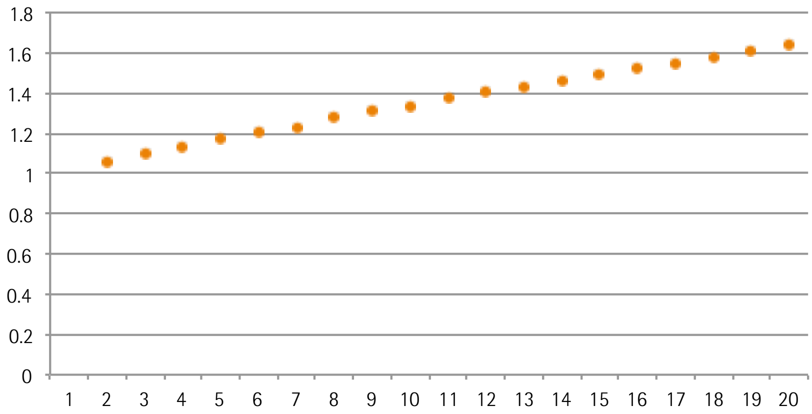


Figure 4: Probabilities of default at time t, generic 20-year economic infrastructure project debt

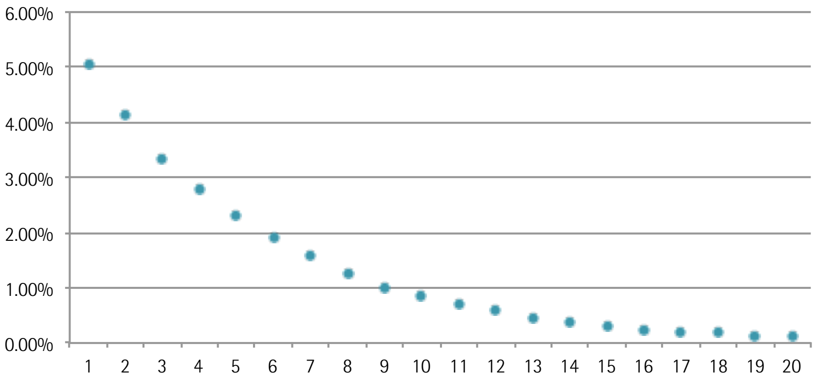


Figure 5: Probabilities of emergence at time t, generic 20-year economic infrastructure project debt

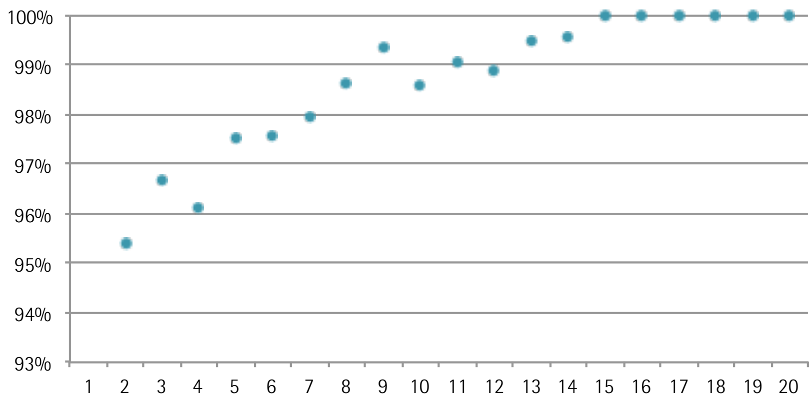


Figure 6: Mapping of DD_t and p_t, generic 20-year economic infrastructure project debt

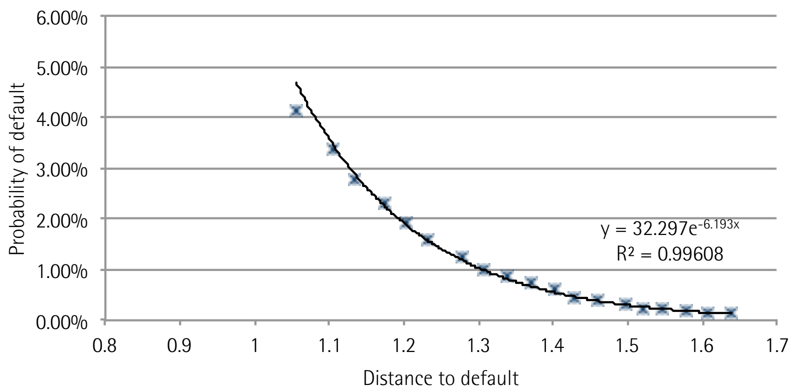


Figure 7: Loss given default at time t, generic 20-year economic infrastructure project debt

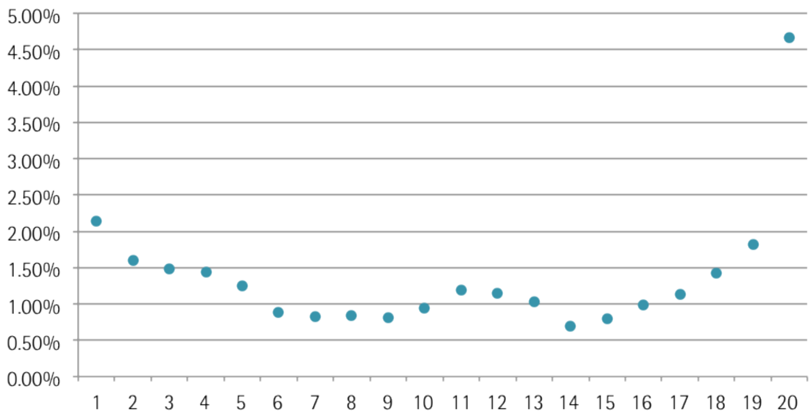
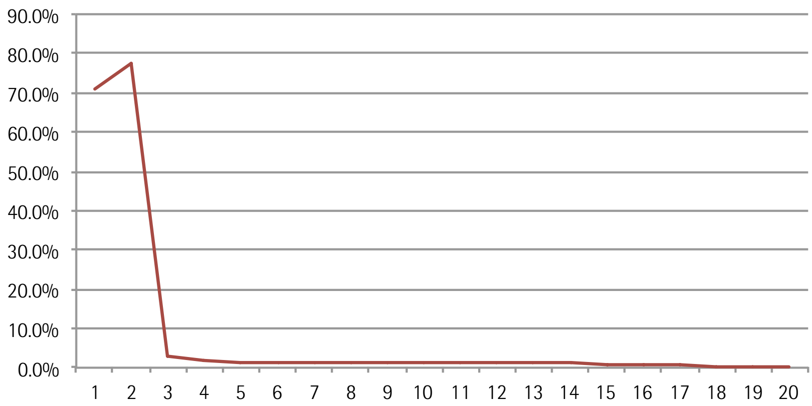


Figure 8: 99.5% Value-at-Risk at time t, generic 20-year economic infrastructure project debt



With a constant and stable DSCR, *by construction*, we obtain a stochastic and random behaviour: distance to default is high, probabilities of default are low and probabilities of emergence are very high; however, there is no systematic effect of the lifecycle since the financing has been structured to compensate for this effect.

As a consequence, as shown on figure 12, there is no statistically significant relationship (i.e. no obvious mapping) between distance to default and probabilities of default. Indeed, the range of values taken by both variables is very limited.

Expected loss given default shown on figure 13 is continuously increasing because, contrary to the previous case, probabilities of default do not trend down with time. Hence, as the debt matures, the size of a one-period loss relative to asset size increases, leading to the computation of higher losses given default.

Finally, value at risk exhibits the same kink than in the previous case, but much later in its life as figure 14 illustrates.

Hence, a constant expected DSCR with constant variance signals infrastructure project debt with a risk profile which is horizon-independent. Moreover, any variability observed across the lifecycle is completely idiosyncratic and therefore diversifiable.

Figure 9: Distance to default, generic 20-year social infrastructure project debt

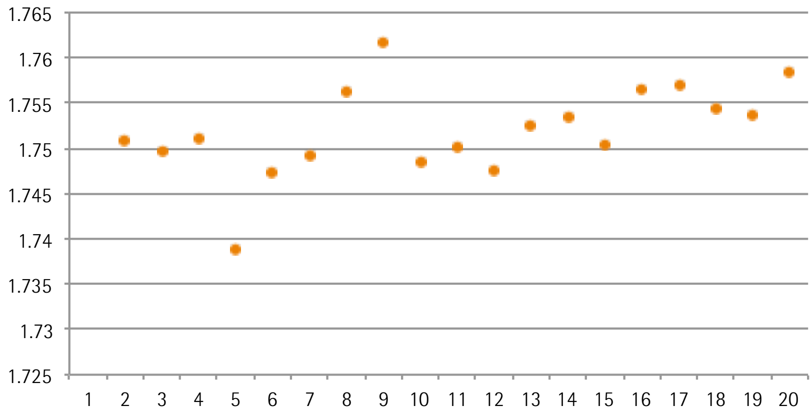


Figure 10: Probabilities of default at time t, generic 20-year social infrastructure project debt

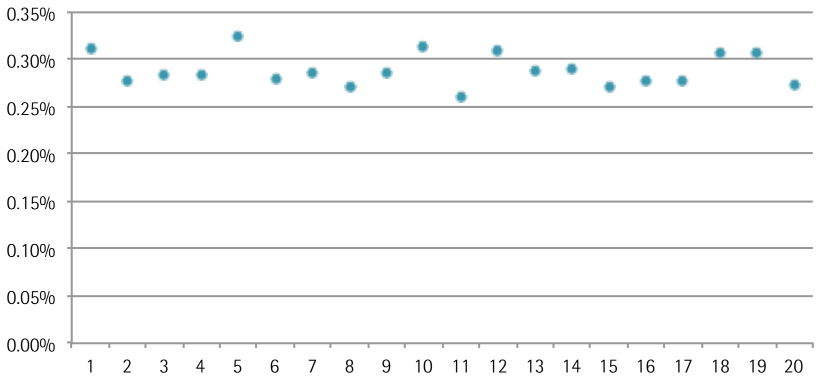
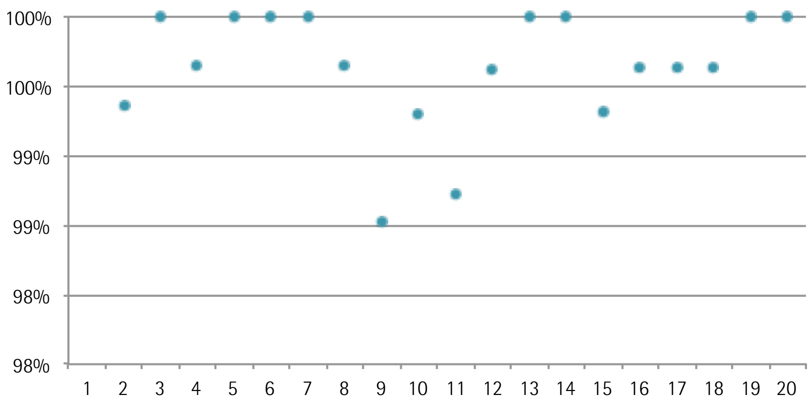


Figure 11: Probabilities of emergence at time t, generic 20-year social infrastructure project debt



Finally, we note that Moody's (2013) also reports very low and constant average probabilities of default (around 0.5%) for "PPP/PFI" project debt, which are not dissimilar to our ideal-type social infrastructure project with a constant DSCR across the lifecycle.

Figure 12: Mapping of DD_t and p_t, generic 20-year social infrastructure project debt

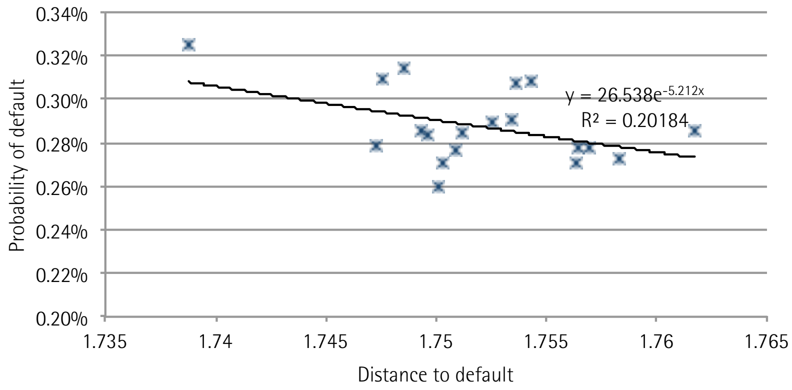


Figure 13: Loss given default at time t, generic 20-year social infrastructure project debt

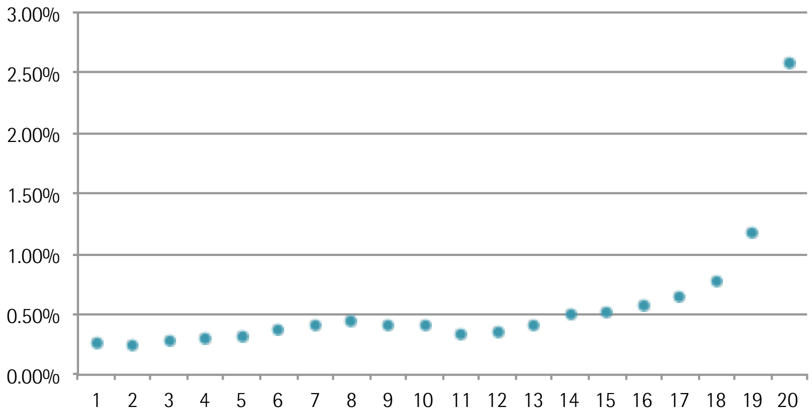
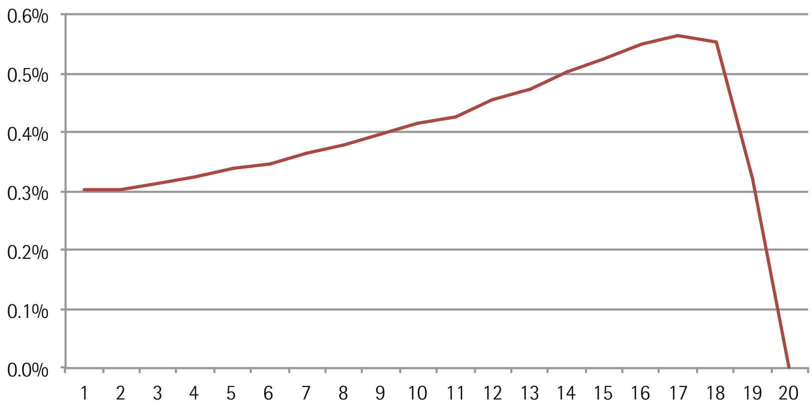


Figure 14: 99.5% Value-at-Risk, generic 20-year social infrastructure project debt



6 Conclusions

6.1 The power of the DSCR

In this paper, we have shown that full knowledge of the distribution of the DSCR in infrastructure project finance is sufficient to characterise the credit risk of infrastructure project debt.

Probably the most appealing aspect of our methodology is to reduce the empirical task of measuring and predicting credit risk to a univariate problem: the estimation of the different moments of $DSCR_t$.

Theoretically speaking, our focus on the DSCR in project finance goes at the heart of the Merton model. The DSCR available in project financing offers the opportunity to measure a dimension of credit risk which typically remains unknown to lenders in the case of classic corporate debt. In limited-recourse project finance, asset values, volatility and the default point can be observed and monitored directly.

Empirically, speaking, our methodology thus avoids relying on proxies (such as inferring asset volatility from listed equity volatility) or ambiguous default points. All the information necessary about asset value, volatility and where the default point lies in included is the distribution of $DSCR_t$.

Operationally, our approach reduces parameter estimation risk considerably.

Indeed, cash flow based models used to derive measures of credit risk produce either sensitivity analyses[21] or Monte Carlo simulations.[22] In effect, both techniques *imply* a distribution of input variables.[23] Each of the input variable distribution is an estimation problem and the opportunity for estimation errors.

Moreover, cash flow sensitivity analyses or simulations require *joint* inputs (e.g. income variability and cost variability), which raises the issues of the relationship between the distribution of each input variable i.e. risk correlations. The number of correlations growth exponentially with the number of model inputs and each pair-wise risk correlation parameter is the opportunity for another set of estimation errors.[24]

Hence, limiting our methodology to the estimation of the distribution of $DSCR_t$ minimises both model and estimation risks. But while it is indeed the most parsimonious approach, the rigorous and robust statistical estimation of $DSCR_t$ is all the more important if our proposed methodology is to produce viable results.

In turn, there is a significant need to collect adequate data and to continue data collection efforts on an ongoing basis, which we discuss next.

6.2 The need to standardise cash flow reporting

Given the potential role of the distribution of $DSCR_t$ for the benchmarking of credit risk in infrastructure project finance, a number of key empirical questions need answering before robust empirical results may be proposed.

21 - Observing the effect of $\pm x$ percentage points variations of one or several input variables on output variables.

22 - Attributing a probability distribution to input variables and observing the distribution of output variables

23 - The shock applied in sensitivity analyses implies its likelihood from the point of view the analyst: choosing to test the impact of a 1% increase of construction costs implies that the scenario under consideration is more likely than if a one percentage point increase in interest rates is considered.

24 - In fact, the majority of cash flow-based risk models ignore risk factor correlations and assume independence between risk factors.

For example, considering a large sample of DSCR observations at each point in the project lifecycle:

1. Is the expected value of $DSCR_t$ explained by fixed factor effects in the cross-section (i.e. risk factors) or do all DSCRs come from the same population?
2. If systematic risk factors explain the expected mean of each $DSCR_t$ population, does this mean value change over time? Does it have a constant variance in time or is there evidence of diffusion?
3. What functional form can be assumed for the distribution(s) of $DSCR_t$ and what are its (their) parameters?
4. Alternatively, what is the functional form of the mapping of distance to default (calculated from $DSCR_t$) and observed probabilities of default. What are the parameters of this mapping?

To answer these questions, a number of data items need to be systematically collected and aggregated. This data can then be used to conduct the necessary analyses[25] and determine the characteristics of the distribution(s) of $DSCR_t$ in project finance.

Comprehensive and systematic data collection with regards to $DCRS_t$ also has wider benefits.

The documentation of the distribution of $DSCR_t$ **contributes directly to the benchmarking of infrastructure debt credit risk**.

It can also help create greater transparency between originators and investors and support the growth of an industry (infrastructure finance) which the policy-maker has repeatedly ear-marked as strategic and instrumental in securing the long-term development and wealth of nations

Finally, and perhaps most instrumentally, the regulation of financial entities providing liquidity to the infrastructure project finance sector, be they banks or institutional investors, can only be improved by a better understanding of the credit dynamics of project finance.

The implementation of the rules established by the Basel Committee or the European Insurance and Occupational Pension Authority for example, may be improved by **considering project financing as a credit category in its own right**, if it can be shown to have unique and distinctive characteristics compared to other credit instruments.

Thus, in the Appendix of this paper (section 7), we propose a simple format of the minimum Cash Flow Reporting requirements necessary to benchmark the credit risk of infrastructure debt.

Combined with the methodology proposed above, the proposed cash flow database is sufficient to estimate the different moments of $DSCR_t$ and fully characterise the credit risk of infrastructure project finance debt.

25 - Variants of ANOVA (analysis of variance) with a focus on panel data analysis implying significant explanatory variables in the cross-section as well as time series effects

7 Appendix: Cash flow reporting requirements

This annex describes the key data items that need to be collected to establish the distribution of $DSCR_t$ in project finance along with its statistical determinants. Table 3 describes the necessary cash flow data to be collected at the SPE level as well as the most relevant risk factors that may be useful in explaining the distribution of $DSCR_t$ and apply the methodology proposed in this paper.

Applied to a large sample of $DSCR_t$ observations, the cash flow data described in table 3 is sufficient to assess the determinants of the distribution of $DSCR_t$ and implement the methodology discussed in this paper.

Apart from the debt cash flows and ratios themselves, only the factors which are expected to have a systematic impact on DSCR variance have to be collected.

Existing empirical research on the determinants of credit spreads in project financing (Blanc-Brude and Strange, 2007; Blanc-Brude and Ismail, 2013b) suggest that a number of risk factors are instrumental in the pricing of infrastructure debt. As a consequence, we expect similar factors to be significant determinants of credit risk and of the distribution of $DSCR_t$. They are the main sources of cash flow volatility in project finance.

Nevertheless, given the limited current knowledge of the significant determinants of credit risk in project finance, a number of variables may also be collected for the purpose of testing the existence of a systematic relationship with the expected value and variance of $DSCR_t$. For example, data about industrial sectors.

The periodicity of the reported cash flow data determines the potential uses of the dataset, including for benchmarking purposes. Considering the illiquidity and long maturities of SPE debt, annual or bi-annual reporting can be considered sufficient.

In the Merton model and the discussion above, the firm enters into a single debt contract, which is repaid gradually and according to its schedule unless there is a default. In effect, a project SPE may use several debt facilities and repay them according to varying schedules. It may also pre-pay or re-finance its debt to benefit from lower interest rates, even though with the rise of institutional investment in infrastructure debt, this practice may become less frequent[26].

Hence, data may also be collected at the debt facility level. Table 4 describes the cash flow data, price and covenant data that may be collected at the credit facility level.

26 - Project finance loans typically do not carry pre-payment penalties and their re-financing can often be described as being a part of the lender/borrower relationship (see Blanc-Brude and Ismail, 2013b, for a discussion). However, institutional investors are attracted by infrastructure debt in part because of its long maturities and are likely to require new loans to be more difficult to re-finance.

Table 3: SPE level data

Data type	Description
Cash flows	1. Base case debt drawdown and service (principal and interest) in each period for all debt facilities 2. Observed debt service cover ratio (DSCR) in each period 3. If known, cash flow available for debt service (CFADS) in each period 4. If known, expected and observed construction-related cash flows
Calendar items	1. Financial close date 2. Contract/concession duration 3. Debt maturity date (all facilities)
Risk factors	*1. Revenue model*
	a. price: indexed & guaranteed, guaranteed or market price b. volume: contracted (public or commercial), part contracted/ part merchant (proportion) or merchant only *2. Input cost risk* (including fuel, labour, technology)
	a. Price (as above) b. Volume (as above) *3. Construction risk*
	a. Construction phase: y/n b. Single fixed-price, fixed-date EPC contract: y/n c. Mega-structure: y/n (e.g. Messina Straight bridge) *4. Counter-party risk*
	a. Off-taker rating b. Public or private
Other factors	1. Total initial senior debt, subordinated debt and equity investment, in the relevant currency 2. Industrial sector (categories congruent with corporate bonds) 3. Country of borrower/issuer 4. Project capacity and units (e.g. million-vehicle km, number of hospital beds, megawatts, &tc.)

Table 4: Loan facility level data

Data type	Description
Cash flows	1. Base case debt drawdown and service for individual debt facilities in each period, by level of seniority. 2. Refinanced facility: y/n. In the event of pre-payment/re-financing, the new base case debt cash flows can replace initial ones in the database.
Pricing	1. Pricing type: fixed rate or fixed spread (over base rate) 2. *Ex ante* spread or rate in each period 3. Swapped benchmark rate: y/n
Covenants	1. Cash sweep: y/n (year) 2. DSCR floor: y/n (level) 3. CFADS/EDBIT conditions

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